ON THE ACCURACY OF MATERIAL WORTH PROFILE ASSUMPTIONS IN HYPOTHETICAL CORE DISASSEMBLY ANALYSES

A Thesis

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This work is dedicated to my parents, and to my friend, Craig H. Greene who taught me the true meaning of being a Christian.

ACKNOWLEDGEMENT

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ABSTRACT

The predominant method of analyzing hypothetical core meltdown accidents is by computer calculations, in which certain assumptions are made about the reactivity worth, and behavior, of the materials in the core. Experiments using the Zero Power Plutonium Reactor at Argonne National Laboratory's site near Idaho Falls, Idaho were conducted which allow verification of some of the assumptions used in computer predictions.

The experiments consisted of a simulated hypothetical core disassembly accident initiated by loss-of-coolant flow. Reactivity worth profile measurements were made in the unperturbed, or reference core, and in subsequent perturbed core configurations.

The main objective of this thesis is to determine if the use of reference core reactivity worth profiles are adequate to predict the reactivity effects of material motion in a severely distorted core. This is accomplished by comparing values of reactivity worth of a zone in the core (where the material movement took place), based on easily measured reactivity worth profiles in the reference (unperturbed) core with zone worth values obtained from measurements in the perturbed core in the hypothetical core disassembly configuration.

In the reference case, the reactivity worth of a portion of the core (drawer) is evaluated from these profile measurements and compared to the worth of the same portion of the core measured by oscillator techniques. The worth of each perturbed drawer is then evaluated from the respective traverses. Finally, the changes in drawer reactivity worth for the progressive perturbations are used to predict the reactivities for the zones of the core in which the sodium voiding, steel slumping, and fuel slumping took place. In general it was found that the use of reference core worth profiles (traverses) can only marginally predict the reactivity effects of material motion in a severely distorted core.

CHAPTER I

Introduction

In a Hypothetical Core Disassembly Accident (HCDA) a likely initiator can be assumed to be a loss of coolant flow. In the case of a Liquid Metal Fast Breeder Reactor, this results in sodium voiding in some regions of the core. Due to this loss of coolant, melting of cladding and consequently melting of fuel is possible. Analysis of this sequence of events is an important part in the fast breeder reactor safety studies.

A major part of the experimental program conducted in the Zero Power Plutonium Reactor Assembly-5 (ZPPR-5) consisted of a simulated Hypothetical Core Disassembly Accident (HCDA) initiated by loss of coolant flow. This was simulated by constructing static representation of the significant steps of such an accident in progress in the ZPPR core. To construct a full-sized model of a reactor core, stainless steel drawers filled with the appropriate materials are placed in a matrix assembly of stainless steel square tubes. Each drawer is an open-topped rectangular box approximately 15 mm by 15 mm x 921 mm long weighing 900 grams when empty. To simulate loss of coolant, clad motion and fuel melting, successive configurations were

constructed to include the progressive voiding of sodium, slumping of steel, and slumping of fuel. This experiment was conducted in two parts, Phase A and Phase B. In Phase A the mocked-up accident sequence was performed in an end-of-cycle configuration, with control elements parked above the core. In Phase B the sequence was then repeated, in reverse, in a beginning-of-cycle configuration, with six of the mock-up control elements inserted. In the initial loading of a reactor core there is an excess of reactivity to allow for the burnup of the fuel during the life of the core. Control rods with absorber materials are inserted at the beginning of cycle to control the excess reactivity, and are gradually withdrawn to compensate for the loss of reactivity due to fuel burnup.

After each configuration was established, sets of radial and axial reactivity traverses were performed. Traverses, or as used in this paper, material worth profiles refer to the reactivity worth of a small-sample as a function of radial or axial position in the core. The data required for the evaluation of small-sample reactivity are the reactor power history and the position of the sample in the core. Starting with the reactor in a steady state the power is recorded while driving the sample in and out of the reactor several times.

Reactivity as a function of time (and position) is then obtained from inverse kinetics, as detailed in Appendix C. If the sample is stopped at positions between "in" and "out" position a reactivity traverse is obtained.

The small-sample reactivity traverses are used to calculate the worth of each perturbed drawer and these, in turn, are used to predict the reactivity worth due to the perturbation of any desired zone in the core. The reactivity worth of all the materials in a drawer were determined by integrating the axial reactivity worth profile of each material over the length of the drawer and summing these values. The radial traverses were used to integrate the worth of a drawer over each zone of interest.

CHAPTER II

The Zero Power Plutonium Reactor Critical Assemblies

The zero power plutonium reactor (ZPPR) is a large split-table machine by which two matrices of horizonally laid, rectangular steel tubes may be brought together remotely. Normally, a core is loaded in the machine with the axis horizontal and half of the reactor in each half of the matrix. Half two is arbitrarily designated as bottom of the reactor. Open-topped steel drawers, normally filled with vertically placed plates, may be placed in the matrices to form a wide variety of reactor assemblies. For the core configuration of materials known as ZPPR-5, each separate matrix was 10-ft. wide, 10-ft. high, and 4-ft. deep, so that the complete matrix was 10 by 10 by 8 ft.

Figures 1A and 1B shows the drawer loading patterns and materials appropriate to ZPPR-5.

Stainless steel clad plutonium uranium molybdenum (Pu-U-Mo) alloy is the principal fuel; similarly, clad sodium simulates the coolant, and unclad U_3O_8, uranium metal, and Fe_2O_3 are added to give the desired material compositions. The Pu-U-Mo alloy consists of a mixture which is 28.5 percent total plutonium, 2.5 percent molybdenum and 69 percent depleted uranium (0.22 weight

Figure 1A. Drawer Loading Patterns for ZPPR-5

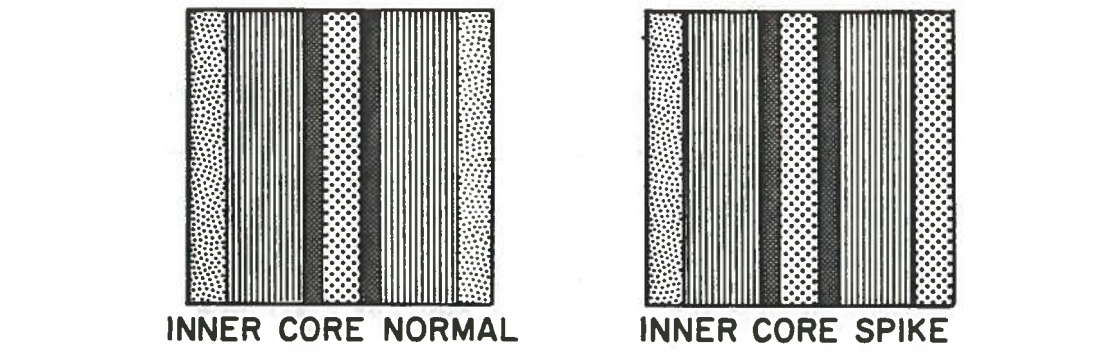
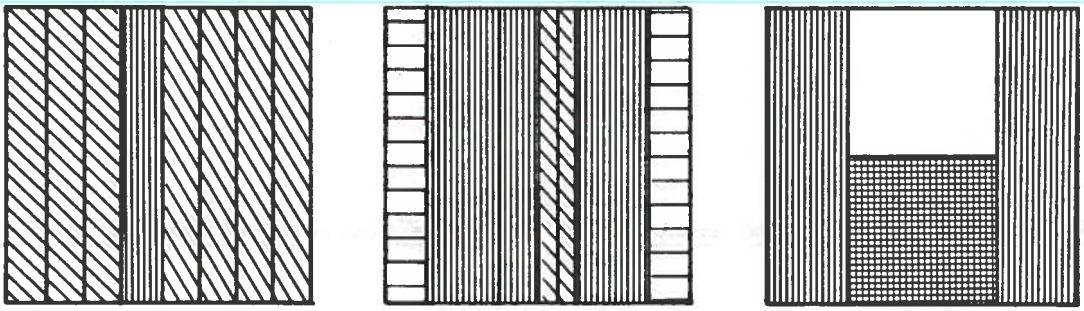
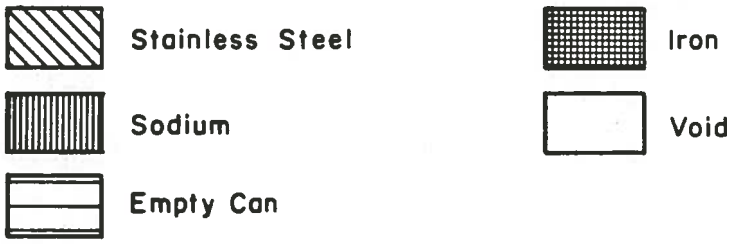


Figure 1B. Drawer Loading Patterns for ZPPR-5





percent ^{235}U). The ZPPR uses this fuel combination because it is necessary to have a prompt negative temperature coefficient based on the Nuclear Doppler effect. These requirements are necessary to assure the safe operation of ZPPR. This requires the initimate mixing of the uranium and the plutonium. The extent to which the reactivity is affected by changes in temperature is described in terms of the temperature coefficient of reactivity

$$\alpha_T = \frac{d\rho}{dT} \qquad (2\text{-}1)$$

where ρ is the reactivity and T is the temperature. In all cases of interest k is close to unity so

$$\alpha_T \simeq \frac{1}{k}\frac{dk}{dT} \qquad (2\text{-}2)$$

and has units of $(\text{degrees})^{-1}$. k is the multiplication factor and is defined as the number of fissions in one generation divided by the number in the preceding generation. An increase in reactor power is reflected first by a rise in the temperature of the fuel, since this is the region where most of the power is generated. The fuel temperature coefficient is defined as the fractional change in k per unit change in fuel

temperature, the fuel temperature coefficient is also called the prompt temperature coefficient.

The prompt temperature coefficient of most reactors is negative, based on the Nuclear Doppler effect.[1] Neutron cross sections exhibit resonances at certain energies, in the heavier nuclei these resonances are due almost entirely to absorption. Since the average neutron flux across the resonance increases with temperature and since the number of neutrons absorbed in the resonance is proportional to the average flux, the resonance absorption increases with increasing temperatures. In our case the absorption is dominated by ^{238}U capture, this decreases k and accounts for the negative value of the prompt temperature coefficient.[1] The molybdenum is added to the fuel alloy to enhance the mechanical properties and to reduce the pyroporosity of the mixture.

A control rod position (CRP) is a region of the assembly which has drawers containing only sodium cans, and which normally extend over the axial core length and axial blanket length. The axial reflector behind the drawer is retained. The material used for reflectors is steel 2 by 2 by 5 inch blocks. When control materials, such as boron carbide are inserted, some of the sodium is displaced.

The mockup control rods (CRs) consist of drawers with absorber material, boron carbide (B_4C), in a region equal in length to the length of the fuel column in a fuel drawer, with the remainder of the drawer normally containing sodium. In a parked control rod (sodium from 0 to 18 inches, boron carbide from 18 to 36 inches, with zero inches measured from the reactor interface), the absorber occupies the region of a drawer corresponding to one axial blanket plus any axial reflector material in the drawer and sodium occupies the region corresponding to the core region in both reactor halves and the other axial blanket. A parked control rod is always ready to be inserted, if needed, which places the absorber material in the core region. When a control rod is fully inserted, the absorber material occupies the core region of both halves and sodium occupies the axial blanket region of both halves.

When a rod is half inserted, absorber material occupies the core region in only one half and of both axial blanket regions. The fixed reactor half is always taken as the "top" of the reactor. The assembly also contains poison safety rods (PSRS) to provide emergency shutdown capabilities.

Assembly 5 was primarily a system to study the neutronic behavior during accident sequences, it was the first of a series of assemblies constituting the

engineering mockup core for the Clinch River Breeder Reactor. The reactor core was designed with two radial zones, inner and outer core. The outer core had a higher fissile material density to increase the power in this region and flatten the power distribution over the core. The outer core zone enrichment was roughly 1.5 times that of the inner core with spiked drawers (drawers with extra fuel) distributed uniformly throughout the core to achieve criticality with the desired core outline. A spike is defined as a column of fuel which replaces a column of diluent material in both reactor halves in a particular matrix position. Spikes are usually added to the core to adjust reactivity.

Small-sample reactivity traverses are obtained which give the reactivity worth profile of a small-sample. This is the reactivity worth of a small-sample as a function of its radial or axial position in the core.

A small-sample may be defined as one that:

(a) Causes only a negligible distortion and depression of the reactor flux at the measuring position;

(b) Has negligible self-shielding, that is, negligible internal flux distortion and depression, and

(c) Has a negligible number of multiple events occurring within it, (i.e., multiple scattering of a neutron before it returns to the reactor or scattering followed by absorption or fission). If these conditions

are satisfied, the reactivity effect caused by the insertion of the sample may be analyzed by first-order perturbation theory. A sample may, in general, be considered to be "small" if the effects described above do not cause more than a one percent deviation in reactivity from the ideal condition since other sources of error are likely to cause uncertainties of this magnitude.[2]

The traverse mechanism and control systems were designed to allow remote control of the positioning of fissile chambers or perturbation samples along the axis or along a line parallel to the axis of the ZPPR reactor.

A perturbation traverse experiment consists of oscillating a sample in and out of the reactor for several cycles. When a perturbation sample reactivity worth traverse is desired, the sample is stopped during the traverse at intermediate points for a constant time interval, using the same sequence in both directions.

The axial/radial traverse mechanism consists of a stepping motor and gear train to move the perturbation sample into and out of the reactor, and a drive motor to rotate the drum magazine holding the samples. A position encoder mounted on the rear end of the traverse mechanism detects rotational movement and gives a digital readout in the control room of the resulting position of the

perturbation sample. The encoder is geared to read in 0.001-inch increments.

The reactivity effect of the oscillating sample is measured by following the power level variations with an ion chamber electrometer connected to a voltage-to-frequency converter, and by following the sample position with the encoder. Neutrons leaking out of the core produce a current in a BF_3 ion chamber. This current is converted to output voltage in an electrometer. The output voltage is connected to a voltage-to-frequency converter which converts the analog signal to a digital signal. The digital signal is stored in a dedicated computer and is the information that goes into the inverse kinetics computer code. Before any measurements are taken the reactor is always kept at constant power to allow delayed neutron precursors to come to equilibrium. The subsequent inverse kinetics analysis of the neutron level variation in the reactor gives the reactivity of the sample as a function of time which can be related to the sample position in the reactor. The solution to the inverse kinetics equation used is described in more detail in Appendix C.[3] A commonly used measure of reactivity is the Inhour unit. One Inhour is that amount of reactivity which will put a reactor on a positive period of one hour after the

delayed neutrons have stabilized. Reactor Power P at time t is given by $P = P_o e^{\alpha t}$ where P_o is the power at time t equal to 0 and α is defined as the inverse of the stable period of the reactor.

At the ZPPR, perturbation samples are 2.173-inch long cylinders with a maximum diameter of 0.42 inches (1.07 cm). One to nine perturbation samples can be loaded in the traverse mechanism and any of the samples can be remotely selected and attached. Perturbation samples can be made from any material of interest and are contained in a stainless steel capsule. Dimensions and compositions of reactivity samples used in ZPPR-5 are shown in Table 1.

Table 1

Description of Reactivity Samples Used
in ZPPR Assembly 5, Phase A

Sample Code	Dimensions, cm Length	O.D.	Sample mass, g	Capsule mass, g	Principal Composition Component	Wt. %
Pu-30	5.519	0.762	38.091	11.600	^{239}Pu	97.204
					^{240}Pu	1.005
					Al	0.95
P240-R	4.775	0.836	13.776	10.222	^{239}Pu	0.904
					^{240}Pu	80.702
					^{241}Pu	0.491
					^{242}Pu	4.003
					O	11.97
U-6	5.519	0.762	46.889	11.463	^{234}U	0.945
					^{235}U	93.192
					^{236}U	0.258
					^{238}U	5.604
Du-6	5.519	0.762	46.427	11.417	^{235}U	0.213
					^{238}U	99.782
SS-1	5.519	0.991	33.635	10.347	Fe	70.92
					Cr	19.20
					Ni	8.70
					Mn	1.42
					Si	0.30
					Cu	0.10

Table 1 (continued)

Sample Code	Dimensions, cm		Sample mass, g	Capsule mass, g	Principal Composition	
	Length	O.D.			Component	Wt. %
B-1	5.519	1.019	4.193	10.521	^{10}B	87.12
					^{11}B	7.38
					O	1.43
					C	0.96

CHAPTER III

Experimental Program

The experimental program began with an end-of-cycle reference configuration known as Phase A.[4] To simulate a reactor core in the last phase of a fuel load, all 12 mockup control rods were in the parked (withdrawn) position, and with sodium in other parts of the channel. In the parked position, all poison segments in the control rods are located in the upper axial blanket. Figure 2 shows a cross-sectional view of the quadrant interface diagram for the reference configuration for the HCDA sequence in Phase A. The rest of the configuration was symmetrical to the section shown. The different sections of the reactor are shown in Figures 1A and 1B. The inner and outer core refer to areas with different fuel loading to maintain a flattened power distribution. Figures 1A, 1B, and 2 show the drawer loading patterns for this configuration. Table 2 shows the heavy-isotope inventory in the different sections of the reactor. The reflector region was constructed with steel blocks. Criticality was achieved by adding spiked drawers to the inner core and outer core regions. The Phase A reference core had a critical mass of 1086.66 kg of fissile plutonium

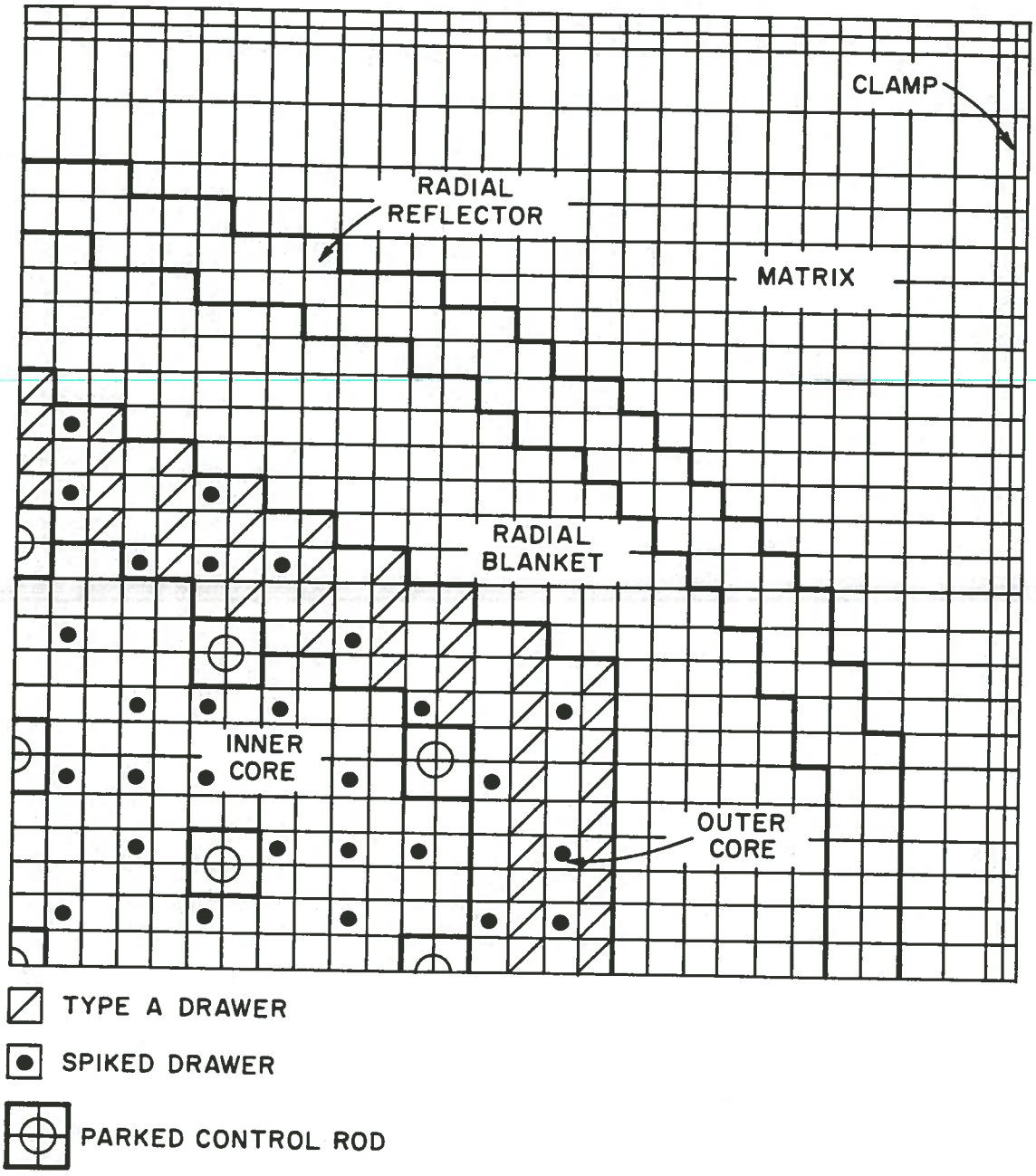


Figure 2. Reference Configuration for the HCDA Sequence in ZPPR Assembly 5, Phase A.

Table 2

Heavy-isotope Loading Summary for ZPPR-5 Phase A Reference Configuration with Parked Control Rods

Material	Mass of Material, kilograms				
	Zone				
	Inner Core	Outer Core	Radial Blanket	Axial Blanket	Total
^{238}Pu	0.304	0.379	--	--	0.683
^{239}Pu	482.234	588.800	--	--	1071.034
^{240}Pu	64.152	78.287	--	--	142.439
^{241}Pu	7.000	8.622	--	--	15.622
^{242}Pu	1.045	1.284	--	--	2.329
Fissile Pu	489.234	597.423	--	--	1086.657
Total Pu	554.735	677.372	--	--	1232.107
Americium	0.663	0.797	--	--	1.460
^{235}U	6.192	4.901	34.032	10.969	56.094
^{238}U	2833.880	2232.085	15613.104	5019.018	25698.087

($^{239}Pu + {}^{241}Pu$). After their reference configuration, the experimental program proceeded through phases of sodium-voiding, steel slump, and fuel slump, with associated small-sample reactivity traverses.

The sodium void pattern expected in a hypothetical loss-of-flow situation in the Clinch River Breeder Reactor was approximated by four radial zones voided sequentially as shown in Figure 3. The assumption was made that zone 3 which is in the more highly enriched outer core region, would be drained of sodium before the inner core is completely voided. To simulate a progressive loss of coolant each of the four radial zones was further subdivided into four axial subzones as shown in Figure 4. These subzones simulated loss of coolant first from the upper axial blanket, then from the top 6 inches of the core, then the next 12 inches, and finally from the bottom 18 inches. The lower axial blanket and the Control Rod Positions (CRPs) were never voided of sodium in any of the zones.

The voiding process consisted of removal of the stainless steel, sodium-filled cans and replacement with empty stainless steel cans in the core region and with stainless steel frames in the blanket regions. During the sodium-voiding sequence the reactor was kept critical by the movement or removal of spikes. The excess

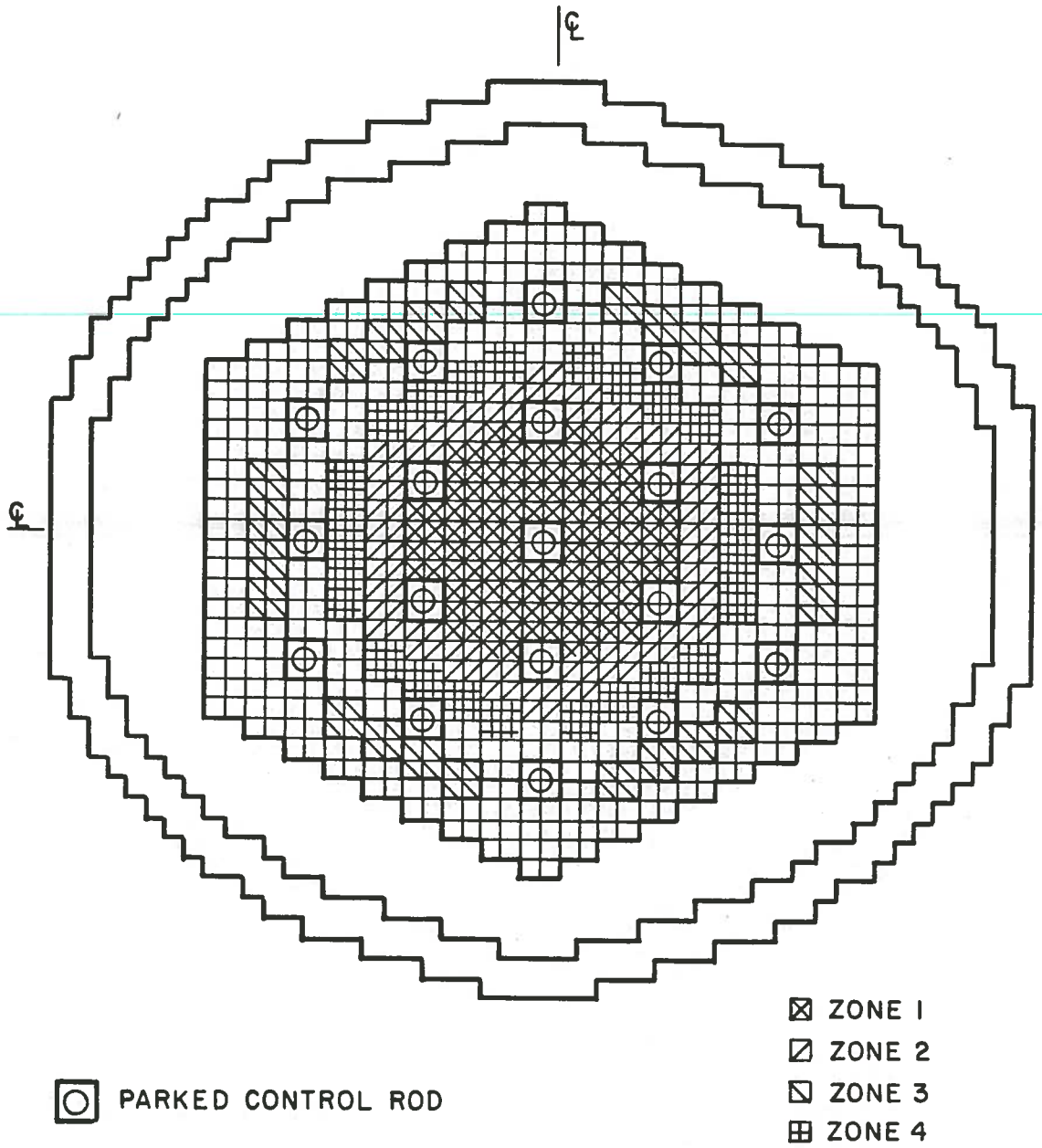


Figure 3. Radial Zones Used in Sodium Voiding Sequences of HCDA Simulation.

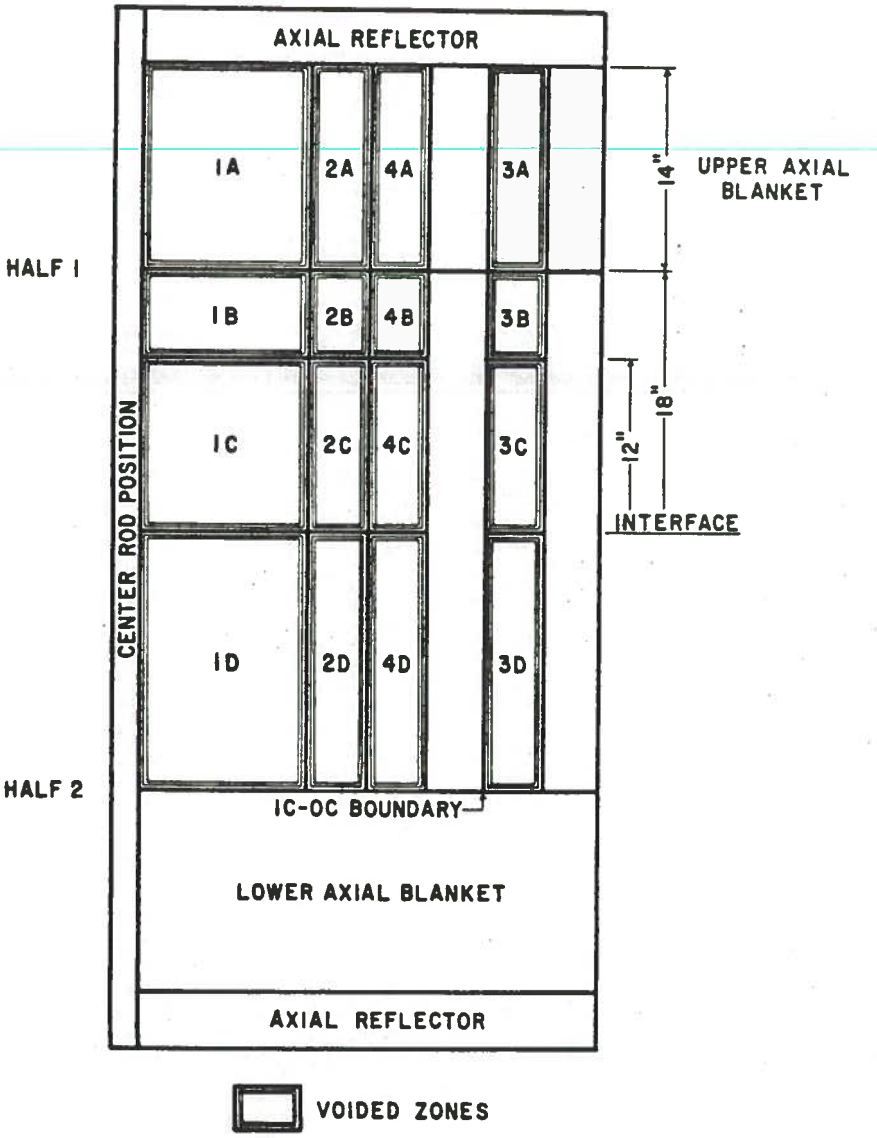
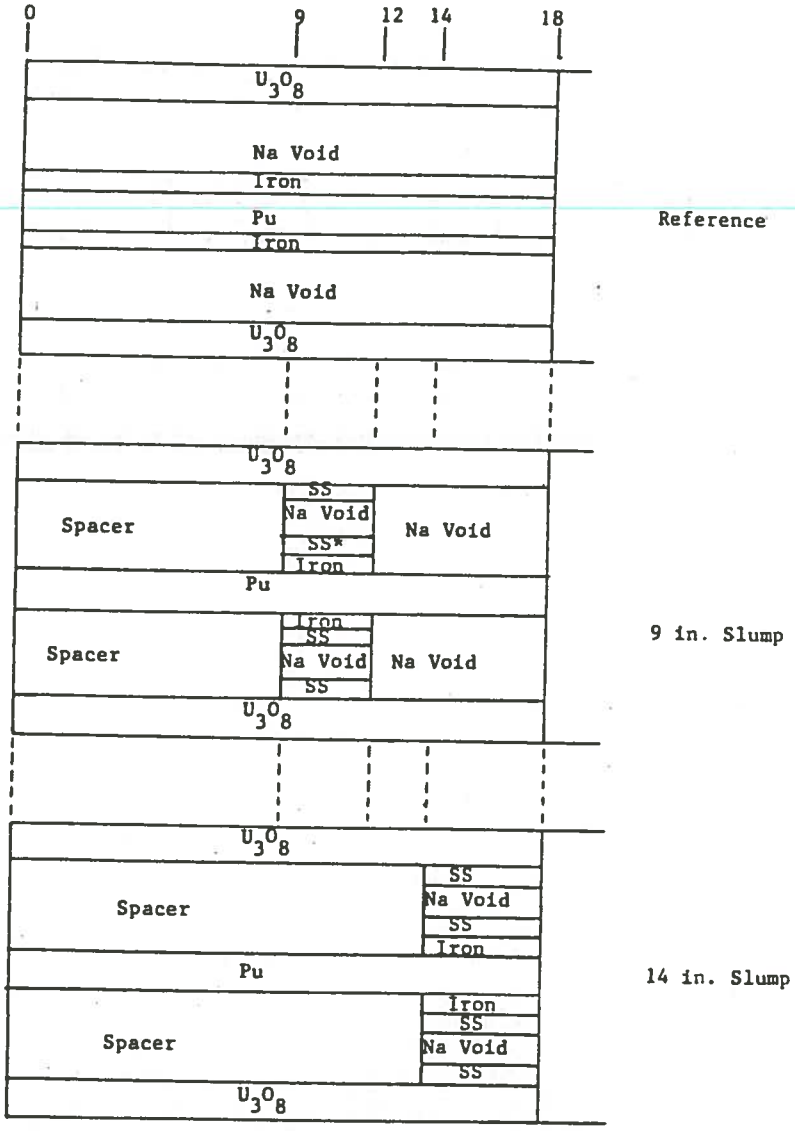


Figure 4. Axial Zones Used in Sodium Voiding Sequence of HCDA Simulation.

reactivity after each step was obtained and adjusted to correct for subcriticality using count rates from BF_3 ion chambers located above the core.

In the steel slumping section of the experiments the inner core was modified to facilitate simulation of clad melting. All steel (or iron) that could be removed by itself was taken out of the low-steel zones and to simulate freezing of the molten metal extra steel was added to smaller adjacent zones. The movable material included the empty sodium cans and the iron in the Fe_2O_3 plates. In order to separate the reactivity worth due to the oxygen from the steel motion, a preliminary step was included in which the 2-inch high Fe_2O_3 plates were replaced with 0.73-inch high iron plates which had the same iron content, but no oxygen. Diagrams of drawer loading for the reference and steel-slumped configuration are shown in Figure 5 and the zone outline in Figure 6.

The sequence followed was steel slumped axially, removed from 0 to 9 inches, and added to 9 to 12 inches from the interface, then slumped further, removed from 0 to 14 inches and added to 14 to 18 inches from the interface. Thin-walled stainless steel tubes were used as spacers to fill the gaps left by removal of the steel as shown in Figure 7.



Iron is 3/4 in. high. *Perforated Plate

Figure 5. Simplified Drawer Masters for Steel Slumping Experiment in ZPPR-5 HCDA Sequence.

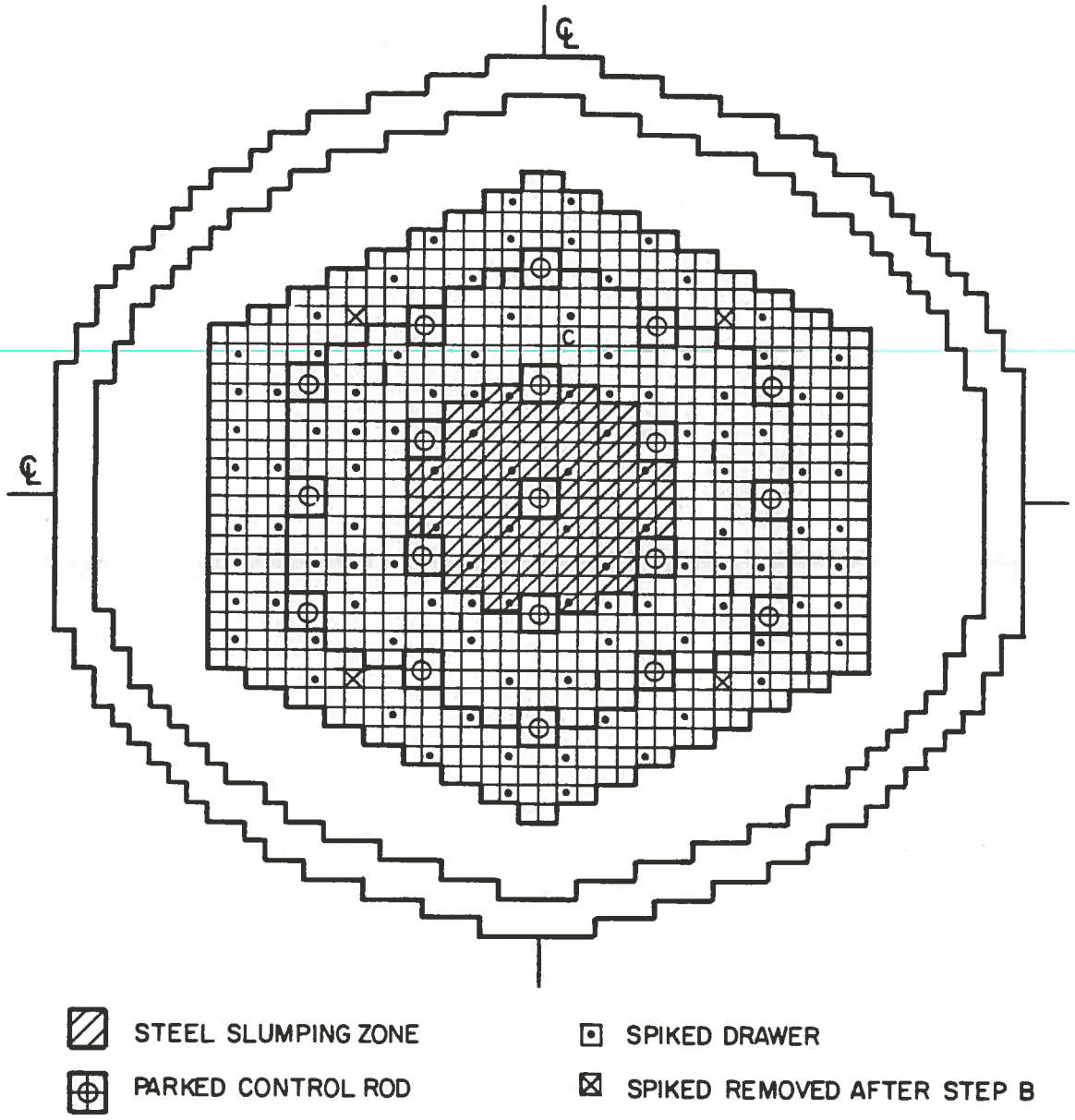


Figure 6. Assembly 5, Phase A. Steel Slumping Zone and Spiking Patterns.

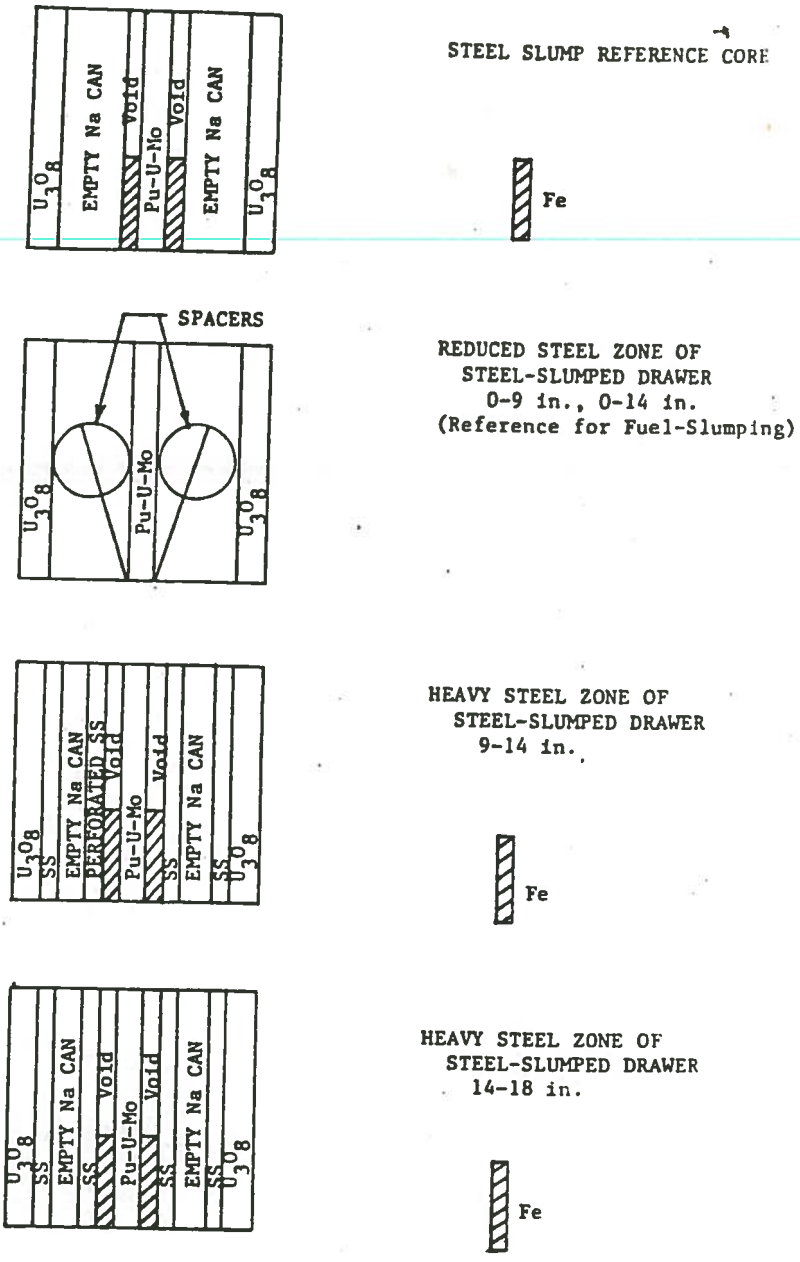


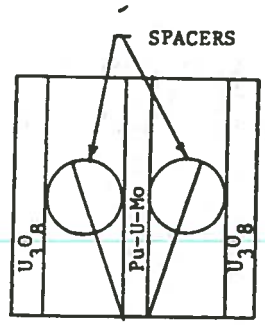
Figure 7. Cross-sectional Views of Standard Inner Core Drawer Loadings for the Steel-Slump Experiments in ZPPR Assembly 5.

To simulate the effect of fuel melting, the only materials in the front 14 inches of the inner core drawers (the fuel-slumped zone) were U_3O_8, ZPPR fuel plates, and the 304SS spacers as shown in Figures 8 and 9. The fuel-slumping experiment was performed in the zone indicated in Figure 10, and included motion of the fuel material (U_3O_8 as well as ZPPR fuel plates) both toward the reactor interface (slump in) and away from it (slump out).

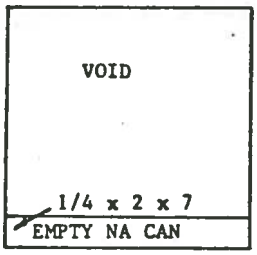
It should be noted that the sodium voiding, steel slumping, and fuel-slumping steps were cumulative; that is, the steel slumping occurred in sodium-voided regions and the fuel slumping occurred in regions of steel slumping and sodium voiding.

The experimental program began with the addition of seven mockup rods to the fuel-slumped Phase A core. This configuration is termed Phase B.[5] A fuel slump-out reference was established with mockup rods fully inserted in the central position and in the outer ring flat positions on the hexagonal core outline. The accident sequence was then followed in reverse, with successive restoration of the fuel slump, steel slump, and sodium void zones. Again, small-sample reactivity traverses were performed at each step.

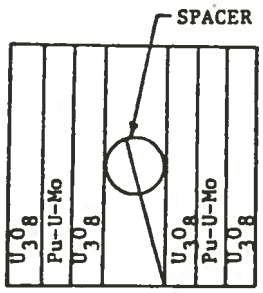
The initial Phase B reference, shown in Figure 11, included the fuel slump-out configuration with which



REFERENCE FOR FUEL-SLUMPING EXPERIMENT
(0-14 in. in core)



FUEL VOIDED ZONE
0-7 in. in slump-out
7-14 in. in slump-in



DOUBLE-FUEL ZONE
0-7 in. in slump-in
7-14 in. in slump-out

Figure 8. Cross-sectional Views of Standard Inner Core Drawer Loadings for the Fuel-Slump Experiments in ZPPR Assembly 5.

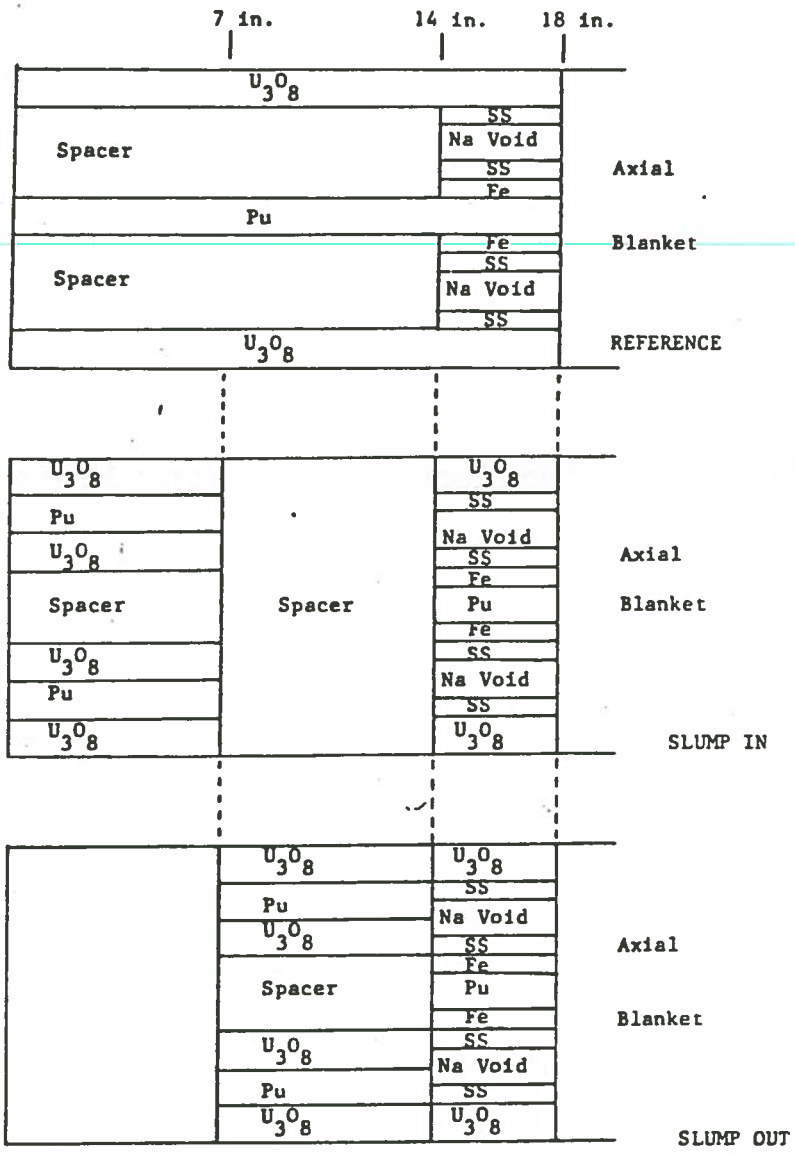


Figure 9. Simplified Drawer Master for Fuel Slumping Experiment in ZPPR-5 HCDA Sequence.

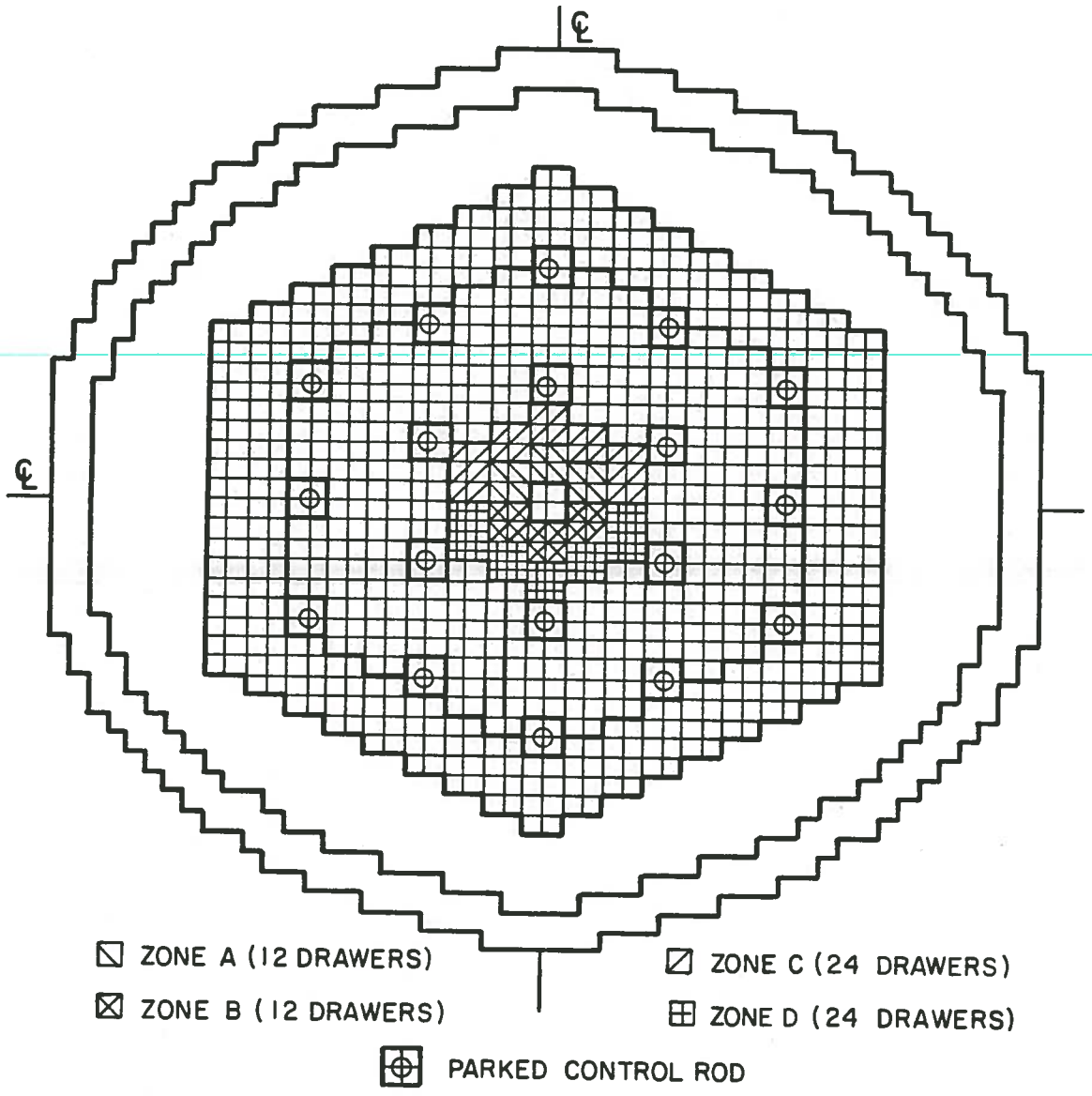


Figure 10. Fuel Slump Zones in the HCDA Configuration of ZPPR Assembly 5, Phase A.

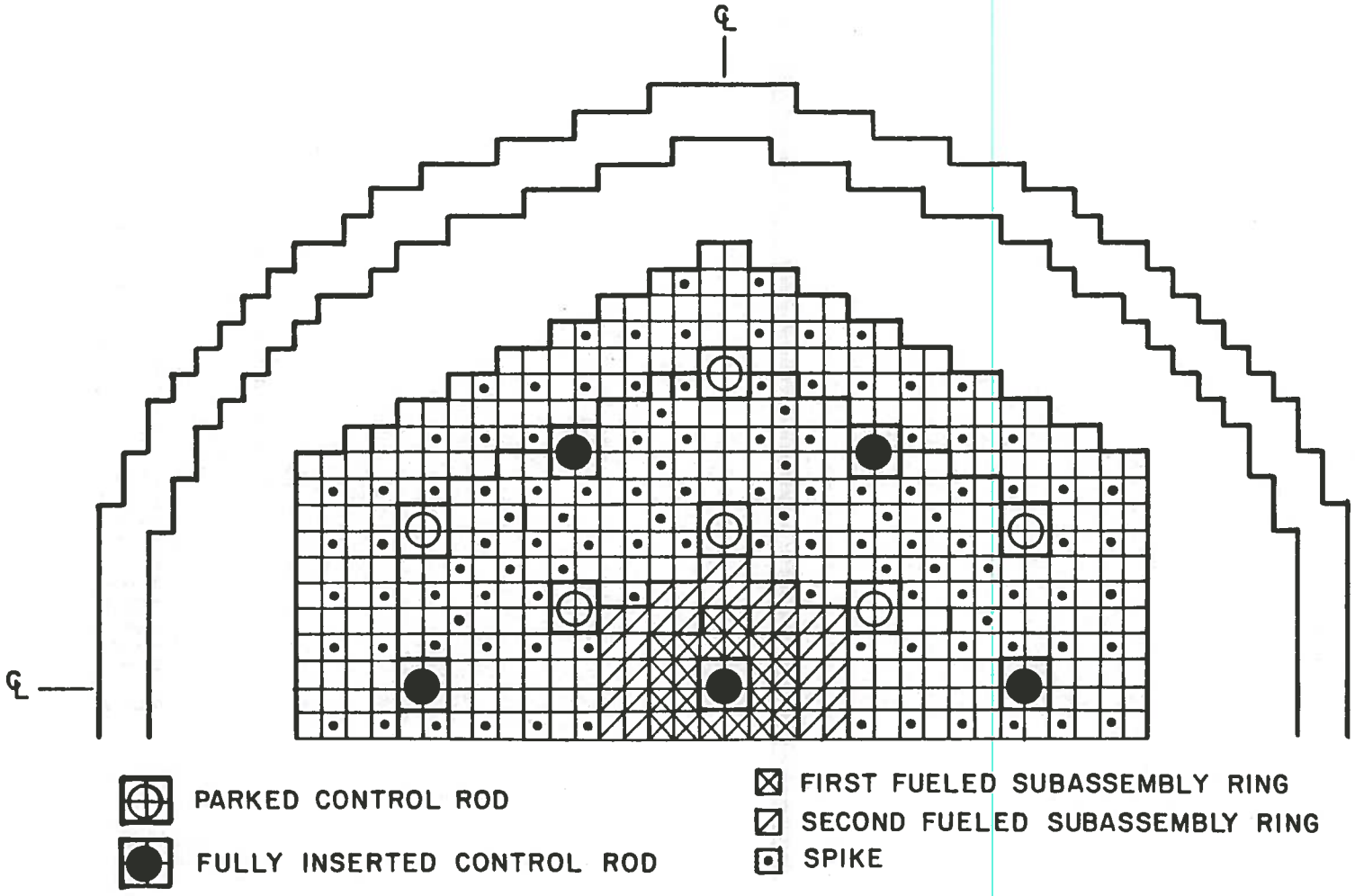


Figure 11. Fuel-Slumped Reference of ZPPR Assembly 5, Phase B.

Phase A had concluded. This was returned to normal fuel configuration in steps so that the equivalent of the first fuel subassembly ring was restored first, then the second. Thus, as an intermediate step, a configuration was achieved with only the second fueled subassembly ring containing slumped fuel, which represents a much more likely circumstance. A massive poison rod was fully inserted in the central position. Each half of each subassembly ring was restored separately to the steel-slumped condition which serves as a reference for the fuel-slumping experiment. The fuel slump experiment involved movement of U_3O_8, steel spacers and ZPPR fuel plates only, as shown in Figures 12 and 8.

After the fuel-slump zone was restored, the equivalent of the first three fueled subassembly rings was left in a condition simulating steel slumping due to clad melting. Drawer loading diagrams for this steel-slumped configuration are included in Figure 12. End views of the steel-slumped drawers are shown in Figure 7. The restored steel-slumped zone is shown in Figure 13.

In the Phase B experiments, the sodium void sequence and pattern were slightly modified relative to Phase A. In Phase A, the outer core void zone was at the closest distance to the center of the core. In

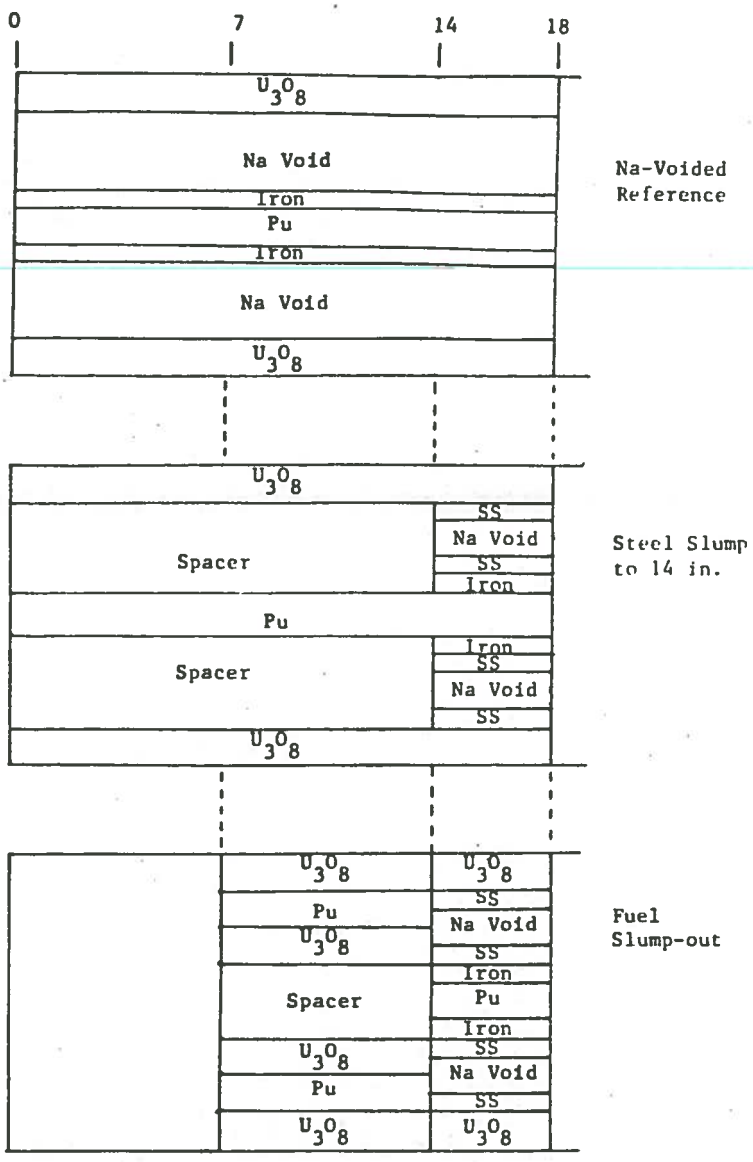


Figure 12. Simplified Drawer Masters for Fuel and Steel Slumping Experiments in the ZPPR-5, Phase B HCDA Sequence.

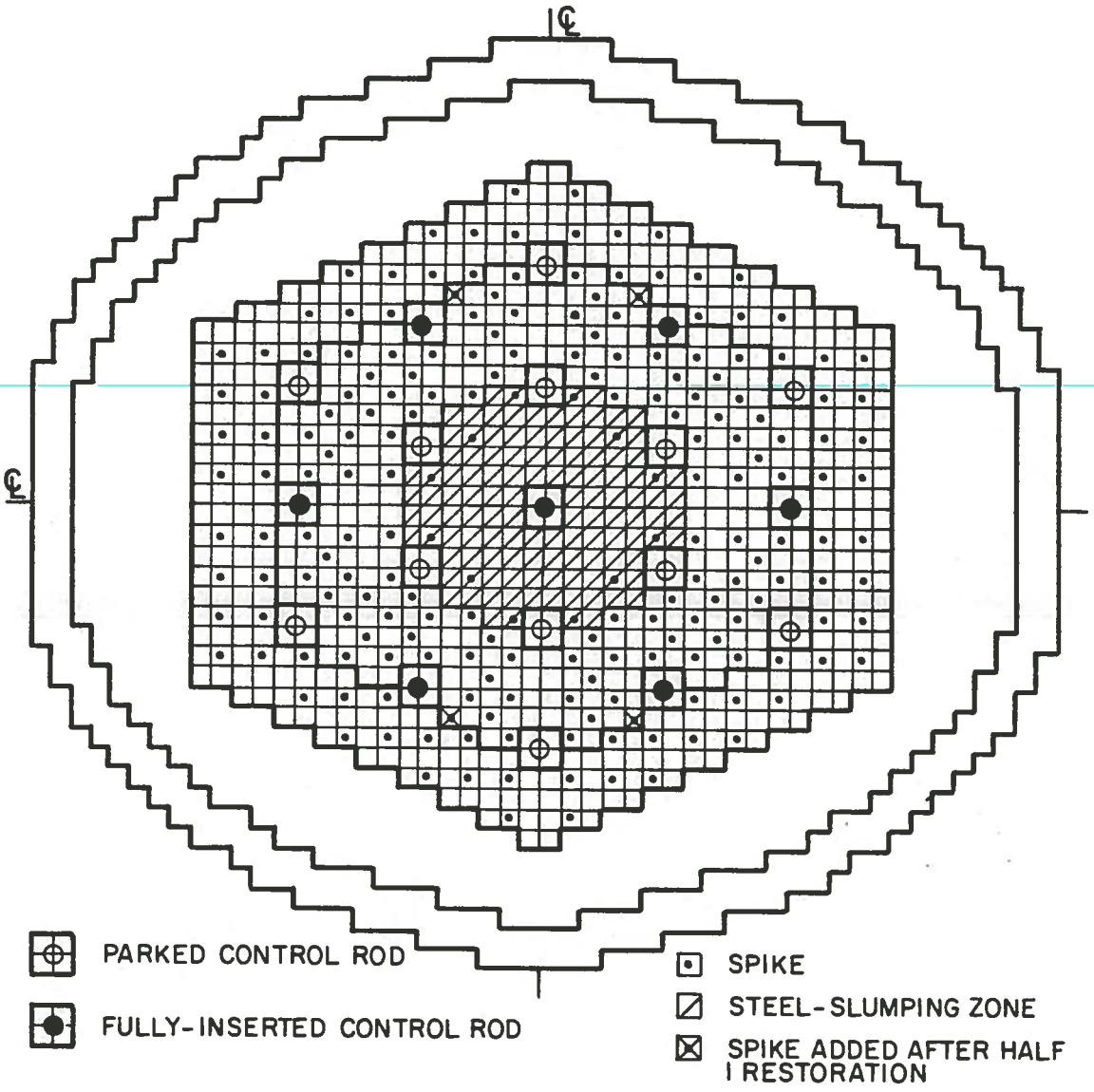


Figure 13. Assembly 5, Phase B, Steel Slumping Zone and Spiking Patterns.

Phase B because mockup control subassemblies were fully inserted in these positions, the voided zones were shifted slightly further from the center of the core. A higher power was expected because the control subassembly in that location was withdrawn. This revised voiding pattern is shown in Figure 14.

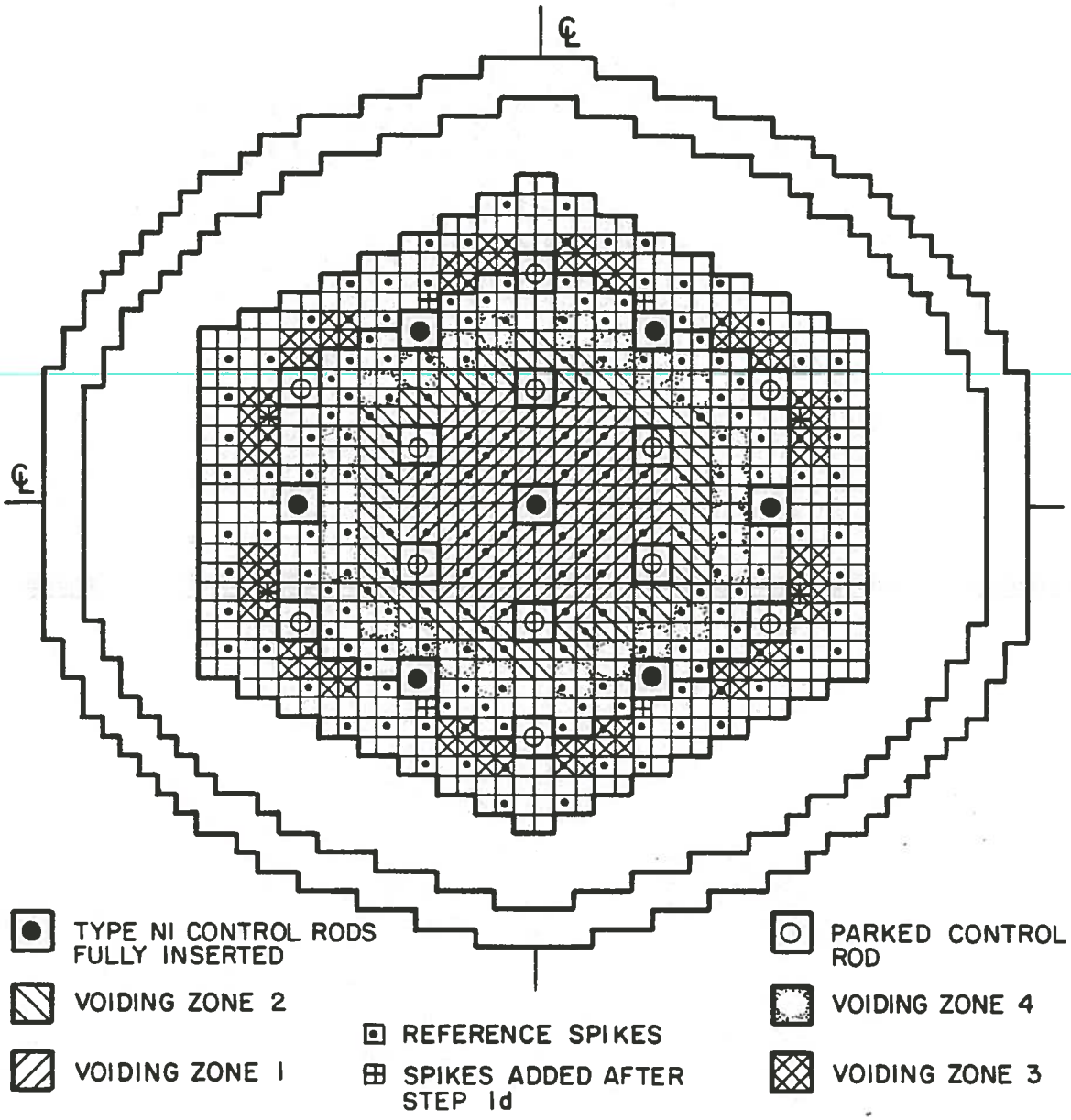


Figure 14. Sodium-voiding Zones in the ZPPR Assembly 5, Phase B HCDA Sequence.

CHAPTER IV

Methodology

The first step in determining the reactivity worth of the different static configurations was to determine the worth of a drawer representative of each configuration. This was accomplished by:

1) Fitting small-sample reactivity worth profiles, obtained in the experimental phase of the program, to a polynomial of up to fifth order,

2) Calculating the mass of each material in a 0.0625-inch thick traverse section of each representative drawer, and

3) Using this information in a computer code to calculate the reactivity worth of a ZPPR drawer.

The purpose of this code is to calculate the reactivity worth of a single ZPPR drawer from a set of axial worth profiles. The worth profiles may be measured or calculated, and should be expressed as a polynomial of up to fifth order, with x equal to 0 at the reactor interface. For more complex profiles, the use of two polynomials over different ranges is allowed. For example, a scattering material may require one polynomial over the front portion of the drawer and a second for the back portion. The range over which a given polynomial is applicable can be specified precisely.

The drawer and its contents is usually divided into a few regions of uniform composition, and masses of each material are specified in each region. The masses are to be in kilograms per 1/16-inch slice of the drawer (1/16 by 2 by 2). The code will then evaluate the polynomial for the value of x corresponding to each slice and each material and sum them accordingly to:

$$\rho_D = \sum_{j=1}^{N} \sum_{i=1}^{576} m_{ij}\rho_j(x_i) \qquad (4\text{-}1)$$

where

ρ_D = reactivity worth of a drawer (inhour);

x_i = x coordinate of ith slice;

$\rho_j(x_i)$ = reactivity per unit mass of jth material at x_i. (This is calculated in the code from the coefficients of the polynomial fit to the worth profile.);

m_{ij} = mass of jth material in ith slice, and

N = number of materials.

There are 576 slices in the 36-inch drawer. Output includes the reactivity contribution of each material as well as the total reactivity. Appendix A lists all the zone worths that were calculated and Appendix D gives the coefficients of the polynomial fit to the worth profile.

The small-sample data were corrected for isotopic composition and for the difference in self-shielding between small-sample and core-drawer configuration were significant. Appendix E includes a listing of the program, input instructions, and sample problems.

Sodium and oxygen were found to contribute significantly to the total reactivity, particularly at the inner core location, but were not measured, because the extremely small worths of the samples make accurate traverses statistically unreliable. It was therefore decided to use calculated worths for these two materials.

Axial traverses were measured in one location, along a single radius from the center of the core. In order to get the reactivity worth of a zone where the material movement took place, the radial traverses of ^{239}Pu were used to determine the radial variations across each zone of the axial traverses. The radial worth of ^{239}Pu was used because it is the principal, and most significant contributor to the worth of the drawer.

The equation used to determine the worth of a zone was

$$\rho_z = \sum_i N_i \frac{\rho(r_i)}{\rho(r_4)} \rho_D \qquad (4\text{-}2)$$

where

r_i = the radius of the ith region;

ρ_z = the reactivity worth of a zone;

N_i = the number of drawers in the zone in the ith region;

$\rho(r_i)$ = the worth of ^{239}Pu in the ith regions;

$\rho(r_4)$ = the position where axial traverses were measured; and

ρ_D = the reactivity worth of one drawer at the location given by r_4.

Appendix B shows all values used for these calculations.

After the total zone worth in each configuration was determined, the worth of the changes in reactivity due to the motion of each material was obtained. The reactivity worth of the fuel-slumped configuration minus the worth of the steel-slumped configuration gives the worth due to the fuel slumping; steel-slumped minus sodium-voided gives the worth due to steel slumping; and the zone worth of the sodium-voided configuration minus the reference (unperturbed core) configuration gives the reactivity worth due to sodium voiding.

For both Phase A (end-of-cycle configuration) and Phase B (beginning-of-cycle configuration), the reactivity worth of a drawer was calculated for the unperturbed drawer, then for each of the three progressive perturbed

cases: the sodium-voided drawer, the steel-slumped drawer, and the fuel-slumped drawer. The worth of each drawer was calculated for both halves of the reactor, because of the nonsymmetry of the perturbation.

To investigate the effectiveness of some of the assumptions made in hypothetical core disassembly analysis, once the worth of the unperturbed drawer was obtained the procedure was repeated three times for all configurations. Three steps were taken in sequence:

a) Each zone worth was calculated from reactivity worth profiles measured in the corresponding configuration;

b) The zone worth of each perturbed configuration was calculated using the traverses measured in the reference configuration; and

c) The zone worth of each perturbed configuration was calculated using the traverses measured in the preceding configuration. (That is, the steel-slumped drawer with sodium-voided traverses was used).

This last method was thought to be a more realistic approach to a real accident sequence, particularly for material motion in a small zone, because it represents the accident sequence while in progress (rather than after completion). This represents the experimental method used at ZPPR to measure reactivity

experimental method used at ZPPR to measure reactivity changes in a hypothetical accident.

CHAPTER V

Results

The results of this work in the form of the reactivity worth of a zone due to the different simulated perturbations are given in Table 3 for Phase A and Phase B.

Table 3

Reactivity Worths in Each Zone

Phase A

	A	B	C	D
Sodium Voided	228.86	96.29	96.29	17.4
Steel Slumped	294.17	129.46	252.46	106.66
Fuel Slumped	27.24	-103.92	45.42	- 8.6

Phase B

	A	B	C	D
Sodium Voided	174.11	144.95	144.95	43.1
Steel Slumped	270.49	177.92	213.71	101.45
Fuel Slumped	- 64.44	-131.06	- 67.76	- 23.1

In Table 3, Column A gives the reactivity worths due to the sodium voiding, steel slumping, and fuel slumping. In each case all zone reactivity worths were calculated from worth profiles measured in the desired perturbation configurations, and in each case the drawer configuration assumed that the desired perturbation was completed. Column B gives the reactivity worths due to each simulated perturbation, but in each case all zone worths were calculated from the reactivity worth profiles measured in the reference configuration. This is the assumption made in some hypothetical core disassembly analyses. Column C gives the reactivity worths due to each simulated perturbation, but for these cases, the reactivity zone worth of each perturbed configuration was calculated using the worth profiles measured in the immediately preceding configuration (i.e., traverses from the steel slumped configuration were used to determine the zone worth due to the fuel slumping.) This represents the case where zone worth values were obtained before each perturbation was completed. This is a more realistic representation of the accident sequence, particularly for a small zone. Column D gives the zone worth due to the sodium voiding, steel slumping, and fuel slumping as actually measured in the experiment.

In Phase A, the reference traverses were measured before the parked rods were installed, therefore the data reported does not reflect the non-symmetry of the HCDA experiment. The use of traverse data from the unperturbed core to predict the worth due to sodium voiding, steel slumping, and fuel slumping (Column B) gives results which are less positive than the use of traverse data from the perturbed core (Column A). It appears from these data that the use of the correct traverses will be the conservative approach for the early stages of the accident sequence.

In comparing values from Column C, obtained from traverses measured before each perturbation was completed, and those from Column A, obtained from traverses measured after each perturbation was completed, to the measured value, it is apparent that regardless of which method is used, we obtained values which are much larger in magnitude than those measured. It appears that for this case the choice of traverses does not make a big difference in the values obtained. The size of the zone involved in the perturbation seems a more significant input parameter for the results obtained.

From the data we can say that in the early stages of the accident sequence, the small-sample technique to predict zone worth over-estimates the reactivity of the system. In the later stages of the

accident sequence, during the fuel slumping, this method definitely underpredicts the reactivity of the material motion.

All the tabulated data used to obtain the results listed above are detailed in Appendices A, B, and D.

The only individual drawer worth that was measured in ZPPR-5 was the reference drawer (unperturbed core) in Axial-1 of Phase A. The measured worth was 9.921 $\pm$ 0.021 cents. The experimentally based reactivity worth deduced from the small-sample integration technique is 9.001 $\pm$ 0.0665. The measured to predicted ratio is 1.1023 $\pm$ 0.0085. The measured to predicted ratio obtained here are in good agreement with the same ratios measured for fissile materials in this core. This result seems to indicate that the use of easily measured small-sample reactivity worth traverses is a good technique for predicting the worth of a drawer in the ZPPR.

Contributions to the total uncertainty in the worth prediction are due to uncertainties in the small-sample traverse measurements and to systematic errors in the small-sample integration technique using a limited number of reactivity shapes. The following pages show the calculations which were done for the Phase A reference drawer to determine the worth of a drawer. Also,

calculations used in obtaining the reactivity worth of a zone due to sodium voiding.

Tables 4 through 9 list the small-sample axial traverses. Reactivities are given as a function of sample position in the core from the interface of the reactor.

In these tables ρ_i is the average value over all experimental measurements determined in each position, and is given by

$$\overline{X} = \frac{\Sigma x_i}{N} \qquad (5\text{-}1)$$

where X_i are the individual values and N is the total of values. The uncertainty σ_i is given by

$$\sqrt{\frac{\Sigma (\overline{X} - X_i)^2}{N(N-1)}} \qquad (5\text{-}2)$$

An approximation of the uncertainty of the reactivity worth due to each material in the drawer is given by the worth of the sample calculated over the desired section of the drawer times the fraction σ_{ave}/ρ_{ave}. All reactivities are given in cents.

Table 4

^{239}Pu Axial Traverse in the Reference Configuration

Sample Position (inches)	Reactivity (inhours/kg) ρ_i	σ_i
1.57	124.2515	± .0784
4.00	119.3862	± .1026
6.00	111.5613	± .1221
8.00	101.4087	± .0913
10.00	89.2959	± .1001
12.00	75.9970	± .1142
14.00	62.2141	± .1276
16.00	49.4035	± .0917
18.00	37.5807	± .1065

$$\sigma_{ave} = [\sum_{i}^{9} (\frac{\sigma_i}{N})^2]^{1/2} \qquad \text{where (N) = 9}$$

$$\sigma_{ave} = [1.22237 \times 10^{-3}]^{1/2} = 0.03496$$

$$\rho_{ave} = \sum_{i}^{9} \frac{\rho_i}{N} \qquad \text{where N = 9}$$

$$\rho_{ave} = \frac{771.0986}{9} = 85.67766$$

$$\frac{\sigma_{ave}}{\rho_{ave}} = 0.00041$$

Worth of Pu calculated over 18 inches $\times \frac{\sigma_{ave}}{\rho_{ave}}$

= uncertainty of the ^{239}Pu worth

$$14.391 \times 0.00041 = 0.0059$$

Table 5

^{240}Pu Axial Traverse in the Reference Configuration

Sample Position (inches)	Reactivity (inhours/kg) ρ_i	σ_i
1.57	12.9834	± .1476
4.00	11.3200	± .1608
6.00	11.7885	± .1808
8.00	10.4044	± .1731
10.00	9.6449	± .1792
12.00	8.1566	± .1681
14.00	7.1549	± .1698
16.00	5.6020	± .1655
18.00	3.6933	± .1717

$\sigma_{ave} = (3.16504 \times 10^{-3})^{\frac{1}{2}} = 0.05626$

$\rho_{ave} = 80.748/9 = 8.972$

$\sigma_{ave}/\rho_{ave} = 0.0063$

Worth due to the Pu-240 calculated over 18 inches = 0.18914

$$0.18914 \times 0.0063 = 0.00119$$

Table 6

^{235}U Axial Traverse in the Reference Configuration

Sample Position (inches)	Reactivity (inhours/kg) ρ_i	σ_i
1.57	88.1998	± .0688
4.00	84.9952	± .0771
6.00	79.3963	± .0824
8.00	72.3582	± .0829
10.00	64.1635	± .0975
12.00	54.9875	± .0691
14.00	45.8483	± .0765
16.00	37.5493	± .0698
18.00	30.1901	± .0804
20.00	24.4626	± .0777
22.00	18.5752	± .0773
24.00	13.8512	± .0808
26.00	10.1331	± .0840
28.00	7.0889	± .0835
30.00	4.6137	± .0784
32.00	3.5159	± .0727
34.00	2.5895	± .0841
36.00	1.7922	± .0675

$\sigma_{ave} = (3.4395 \times 10^{-4})^{\frac{1}{2}} = 0.01855$

$\rho_{ave} = 644.3105/18 = 35.79503$

$\sigma_{ave}/\rho_{ave} = 0.00052$

Worth due to U-235 calculated over 36 inches = 0.18010

$0.18010 \times 0.00052 = 0.00009$

Table 7

^{238}U Axial Traverse in the Reference Configuration

Sample Position (inches)	Reactivity (inhours/kg) ρ_i	σ_i
1.57	-6.2696	± .0373
4.00	-5.6213	± .0442
6.00	-5.3445	± .0476
8.00	-4.8086	± .0434
10.00	-3.9863	± .0467
12.00	-2.8366	± .0493
14.00	-2.3076	± .0492
16.00	-1.3079	± .0498
18.00	- .5680	± .0470
20.00	.0029	± .0452
22.00	.0160	± .0454
24.00	- .0221	± .0508
26.00	.0445	± .0463
28.00	.0942	± .0499
30.00	- .0640	± .0496
32.00	.1236	± .0474
34.00	- .0167	± .0498
36.00	- .0395	± .0469

$\sigma_{ave} = (1.232H\text{-}04)^{\frac{1}{2}} = 0.0111001$

$\rho_{ave} = -32.911S/18 = -1.82842$

$\sigma_{ave}/\rho_{ave} = -.00607$

$-0.00607 \times (-4.1102) = 0.02495$

Table 8

Stainless Steel (first part) Axial Traverse in the Reference Configuration

Sample Position (inches)	Reactivity (inhours/kg) ρ_i	σ_i
1.57	-4.9671	± .0871
4.00	-4.0666	± .0619
6.00	-3.8314	± .0713
8.00	-2.9955	± .0644
10.00	-2.4511	± .0652
12.00	-1.5581	± .0686
14.00	- .7477	± .0669
16.00	- .0145	± .0762
18.00	.5603	± .0673

$\sigma_{ave} = (5.48363 \times 10^{-4})^{\frac{1}{2}} = 0.02342$

$\rho_{ave} = -20.0717/9 = -2.23019$

$\sigma_{ave}/\rho_{ave} = -0.0105$

$-0.77710 \times -0.0105 = 0.00816$

Table 9

Stainless Steel (second part) Axial Traverse
in the Reference Configuration

Sample Position (inches)	Reactivity (inhours/kg) ρ_i	σ_i
20.00	.3073	± .0605
22.00	.1094	± .0773
24.00	-.0033	± .0703
26.00	-.3005	± .0681
28.00	-.3516	± .0749
30.00	-.4653	± .0621
32.00	-.3885	± .0738
34.00	-.6249	± .0753
36.00	-.4644	± .0775

$\sigma_{ave} = (5.65487 \times 10^{-4})^{\frac{1}{2}} = 0.02378$

$\rho_{ave} = -2.1818/9 = -0.24242$

$\sigma_{ave}/\rho_{ave} = -0.09809$

$-0.07238 \times -0.09809 = 0.0071$

The calculated worth of oxygen and sodium are good in the shape of the worth profile but poor in magnitude. We assume a 10% uncertainty everywhere. So for oxygen

Oxygen 1: $-.75753 \times 0.1 = -0.07575$

$-.75753 \times 0.1 \times -0.07575 = 0.05739$

and

Oxygen 2: $.10919 \times 0.1 = 0.01092$

$.10919 \times 0.01092 = 0.00119$

and for sodium:

Sodium 1: $-0.41689 \times 0.1 = -0.04169$

$-0.41689 \times -0.04169 = 0.01738$

and

Sodium 2: $0.26439 \times 0.1 = 0.02644$

$0.26439 \times 0.02644 = 0.00699$

These calculated uncertainties are listed in Table 10 with the computer calculated values of each material reactivity worth. The measured to predicted ratio calculation is also given.

Table 10

Reactivity Worth of the Reference Drawer in Phase A

Material	Reactivity	Uncertainty
304 SSA	- 0.77710	± 0.0082
304 SSB	- 0.07238	± 0.0071
U-235	0.18010	± 0.0001
Pu-240	0.18914	± 0.0012
U-238	- 4.1102	± 0.0250
Pu-239	14.391	± 0.0059
Sodium 1	- 0.41689	± 0.0174
Sodium 2	0.26439	± 0.0070
Oxygen 1	- 0.75753	± 0.0574
Oxygen 2	- 0.10919	± 0.0012
Total	9.0001	± 0.0665

$$\sigma_T = \sqrt{\Sigma(\sigma_i)^2} = \sqrt{4.42687 \times 10^{-3}} = 0.0665347$$

Measured to predicted ratio

$$\frac{9.921 + 0.021}{9.0001 \pm 0.067} = \frac{9.921}{9.0001} \pm \frac{9.921}{9.0001}\sqrt{\left(\frac{0.021}{9.921}\right)^2 + \left(\frac{0.067}{9.0001}\right)^2}$$

$$= 1.1023 \pm 0.00853$$

$$\rho_D = 18.0002 + \sqrt{(0.0665)^2 + 0(.0665)^2}$$

$$= 18.0002 \pm 0.0941$$

Because all the traverse data have approximately the same uncertainty, to a first order of approximation, the uncertainty in all other drawers will be about the same as the reference drawer.

Table 10 (continued)

Calculations to determine the uncertainty in the value of zone worth

The total reactivity worth of the reference drawer for both halves of the reactor is given by

$$\rho_D = 18.0002 \pm 0.0941$$

The worth of a zone is given by:

$$\sum_n \rho_D N_i \rho(r_i/r_4). \qquad (5\text{-}3)$$

Since ρ_D and ρ_{r_4} are constant, so the expression becomes

$$\frac{\rho_D}{\rho_{r_4}} \sum_n N_i \rho_{r_i} \qquad (5\text{-}4)$$

Variables in these two equations are described in Chapter IV. For Phase A, reference the values of Pu-239 radial traverses are given in Table 11. The Phase A sodium voided configuration radial traverses are given in Table 12.

Table 11

Radial Traverses for Pu-239

Sample Position (inches)	N_i	Reactivity (inhours/kg) ρ_i	$N_i\rho_i$
r_2 = 3.26	8	128.6160 ± .1178	1028.9280 ± 0.9424
r_3 = 5.44	16	127.8669 ± .1249	2045.8704 ± 1.9984
r_4 = 7.61	24	124.1522 ± .1241	2979.6528 ± 2.9784
r_5 = 9.79	28	119.7732 ± .1121	3353.6496 ± 3.1388
r_6 = 11.96	28	113.9848 ± .1180	3191.5744 ± 3.304
r_7 = 14.14	16	108.7459 ± .1214	1723.9344 ± 1.9424

$$\sum_n N_i\rho_i = 14323.579 \pm 6.19770$$

$$= 14323.579 \pm 6.1977$$

So the worth of a zone

$$= \frac{18.0002 + 0.0941}{124.1522 \pm .1241} (14323.579 \pm 6.1977)$$

$$= \frac{(18.0002)(14323.579)}{124.1522} \pm \frac{(18.0002)(14323.579)}{124.1522}$$

$$\sqrt{\left(\frac{0.0941}{18.0002}\right)^2 + \left(\frac{6.1977}{14323.579}\right)^2 + \left(\frac{.1241}{124.1522}\right)^2}$$

$$= 2076.7033 \pm 11.08956$$

Table 12

^{239}Pu Radial Traverses for Phase A Sodium-Voided Reference Configuration

$\rho_D = 19.8092 \pm 0.0941$

Sample Position (inches)	N_i	Reactivity (inhours/kg) ρ_i	$N_i \rho_i$
$r_2 = 3.26$	8	$113.4405 \pm .1295$	907.5240 ± 1.0360
$r_3 = 5.44$	16	$112.7849 \pm .1207$	1804.5584 ± 1.9312
$r_4 = 7.61$	24	$110.8294 \pm .1182$	2659.9056 ± 2.8368
$r_5 = 9.79$	28	$107.7646 \pm .1244$	3017.4088 ± 3.4832
$r_6 = 11.96$	28	$104.0248 \pm .1555$	2912.6944 ± 4.354
$r_7 = 14.14$	16	$99.8336 \pm .1419$	1597.3376 ± 2.2704

$\sum = 12899.4290 \pm 7.00678$

so the worth of this zone is

$$= \frac{19.8092 + 0.0941}{110.8294 \pm 0.1182} (12899.4290 \pm 7.0068)$$

$$= \frac{19.8092 \times 12899.4290 + 19.8092 \times 12899.4290}{110.8294}$$

$$\sqrt{\left(\frac{0.0541}{19.8092}\right)^2 + \left(\frac{7.0068}{12899.4290}\right)^2 + \left(\frac{0.1182}{110.8294}\right)^2}$$

$$= 2305.5919 \pm 11.29458$$

Table 12 (continued)

The zone worth due to sodium voiding is

$$(2305.5919 \pm 11.2946) - (2076.7033 \pm 11.08563)$$

$$= 2305.5919 - 2076.7033 \pm \sqrt{(11.2946)^2 + (11.08563)^2}$$

$$= 228.8886 \pm 15.8259$$

All reactivities shown above are given in cents.

CHAPTER VI

Conclusions

The principle objective of this work is to determine if the use of reactivity worth profiles, measured in a unperturbed core, are adequate to predict the reactivity effects of material motion in a severely distorted core.

The method is used to predict the worth of a single drawer and the results are compared to the worth previously measured by the oscillator technique in Chapter 5 of this thesis. The comparisons indicate that the use of small-sample reactivity traverses is an acceptable technique for predicting the worth of a drawer in the ZPPR.

The reactivity worths due to sodium-voiding, steel slumping, and fuel slumping are calculated from the worth profiles measured in the desired perturbation configuration. This configuration represents the case in which all material movement is assumed to be complete.

The next step is to predict the reactivity worth of each stage of the simulated core disassembly accident using the reactivity profiles measured in the unperturbed core. Before fuel motion takes place, this method

predicts reactivity values that are higher than the measured zones worth. Once the fuel has moved, the predicted reactivity is more negative than the measured value by a factor of five.

To provide a more realistic representation of an accident sequence, the reactivity worths of each perturbation are calculated using worth profiles measured in the immediately preceding configuration. This represents the situation in which zone worth values are obtained before each material movement was completed.

It is found that in all approaches the absolute value of the calculated results are larger than those measured by factors ranging from 1.76 to 5.67. The choice of reactivity traverses used in the calculations does not make a significant difference in the values of zone worth obtained. The most significant parameter affecting worth is the size of the zone involved in the perturbation.

Therefore the method of using reactivity profiles as measured in an unperturbed core can be used to predict the worth of materials in the analysis of a hypothetical core disassembly accident, prior to fuel motion. This method is not conservative (that is, it predicts lower reactivities) in the accident after the fuel has been displaced.

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APPENDIX A

Reactivity Worth Calculated for a Typical Drawer in Each Core Configuration

This appendix lists the reactivity worth calculated for a typical drawer in the reference, and each perturbed configuration. Values are also given for the reactivity contribution of each material in the drawer. The information is presented for Phase A axials 1 and 2, and Phase B axials 1 and 2.

Data are given to four significant figures because of the accuracy of the measurements and the necessity of minimizing round-off errors in subsequent calculations.

This appendix lists Type 304 stainless steel as 304 SS. The letter (A or B) or number (1 or 2) that follows this designation for these steels, sodium, and oxygen represents the two segments of the curve used to describe the reactivity profile of each material. These segments are input separately into the computer to accomplish the analysis of the data.

Phase A

(All reactivities in cents)

Axial-1

Material	Reference Reactivity	Sodium Voided Reactivity	Steel Slumped Reactivity	Fuel Slumped Reactivity
304 SSA	- 0.7771	- 0.3453	0.1847	0.2054
304 SSB	- 0.0724	0.2067	0.1197	0.1710
U-235	0.1801	0.1802	0.1802	0.1429
Pu-240	0.1891	0.8899	0.1894	0.2488
U-238	- 4.1102	- 2.4582	2.1780	1.8827
Pu-239	14.391	12.619	13.085	12.543
Sodium 1	- 0.4169	0.00	0.00	0.00
Sodium 2	0.2644	0.0002	0.0002	0.0002
Oxygen 1	- 0.7575	- 0.4858	- 0.4858	- 0.2732
Oxygen 2	0.1092	0.0974	0.0974	0.0954
Totals	9.0001	10.0004	11.192	11.260

Phase A

(All reactivities in cents)

Axial-2

Material	Reference Reactivity	Sodium Voided Reactivity	Steel Slumped Reactivity	Fuel Slumped Reactivity
304 SSA	- 0.7771	- 0.5491	- 0.0267	0.2025
304 SSB	- 0.0724	- 0.0163	- 0.0758	0.0469
U-235	0.1801	0.1802	0.1802	0.1710
Pu-240	0.1891	0.1890	0.1927	0.2674
U-238	- 4.1102	- 3.3763	3.0579	2.9238
Pu-239	14.391	13.709	14.146	13.942
Sodium 1	- 0.4169	0.00	0.00	0.00
Sodium 2	0.2644	0.0574	0.0574	0.0574
Oxygen 1	- 0.7575	- 0.4857	- 0.4856	- 0.2736
Oxygen 2	0.1092	0.8974	0.0974	0.0954
Totals	9.0001	9.8052	11.035	11.180

Phase B

(All reactivities in cents)

Axial-1

Material	Reference Reactivity	Sodium Voided Reactivity	Steel Slumped Reactivity	Fuel Slumped Reactivity
304 SSA	- 0.4733	- 0.4715	0.0812	0.0688
304 SSB	0.0512	0.1650	0.0918	0.1006
U-235	0.1507	0.1507	0.1507	0.1254
Pu-240	0.2193	0.2191	0.2196	0.2309
U-238	- 3.0748	- 2.3482	2.0141	- 1.6954
Pu-239	13.062	12.376	13.046	11.281
Sodium 1	- 0.4169	0.00	0.00	0.00
Sodium 2	0.08336	0.0002	0.0002	0.0002
Oxygen 1	- 0.7575	- 0.4856	- 0.4856	- 0.2732
Oxygen 2	- 0.1092	0.0974	0.0974	0.0954
Totals	8.9526	9.7029	11.188	9.9334

Phase B

(All reactivities in cents)

Axial-2

Material	Reference Reactivity	Sodium Voided Reactivity	Steel Slumped Reactivity	Fuel Slumped Reactivity
304 SSA	- 0.4927	- 0.5999	- 0.0274	- 0.0574
304 SSB	0.1220	0.1345	0.2329	0.0733
U-235	0.1662	0.1663	0.1663	0.1501
Pu-240	0.2158	0.2156	0.2160	0.2390
U-238	- 3.9116	- 3.1953	- 2.7439	- 2.4183
Pu-239	13.756	13.273	12.428	12.207
Sodium 1	- 0.4169	0.00	0.00	0.00
Sodium 2	0.08336	0.0574	0.0574	0.0574
Oxygen 1	- 0.7575	- 0.4856	- 0.4856	- 0.2736
Oxygen 2	0.1092	0.0974	0.0974	0.0954
Totals	8.8743	9.6636	9.9415	10.073

Phase A

With Reference Traverses

(All reactivities in cents)

Axial-1

Material	Sodium Voided Reactivity	Steel Slumped Reactivity	Fuel Slumped Reactivity
304 SSA	- 0.7657	- 0.2586	- 0.1918
304 SSB	- 0.0762	- 0.07618	- 0.0401
U-235	0.1802	0.1802	0.1610
Pu-240	0.1890	0.1894	0.1667
U-238	- 4.1102	- 4.1102	- 3.2229
Pu-239	14.375	14.375	12.323
Sodium 1	0.00	0.00	0.00
Sodium 2	0.0002	0.0002	0.0002
Oxygen 1	- 0.4858	- 0.4858	- 0.2732
Oxygen 2	0.0974	0.0974	0.0954
Totals	9.4035	9.9111	9.0178

Phase A

With Reference Traverses

(All reactivities in cents)

Axial-2

Material	Sodium Voided Reactivity	Steel Slumped Reactivity	Fuel Slumped Reactivity
304 SSA	- 0.7657	- 0.2586	- 0.1918
304 SSB	- 0.2703	- 0.2703	- 0.0272
U-235	0.1802	0.1802	0.1610
Pu-240	0.1890	0.1894	0.1667
U-238	- 4.1102	- 4.1102	- 3.2229
Pu-239	14.375	14.375	12.323
Sodium 1	0.00	0.00	0.00
Sodium 2	0.0574	0.0574	0.0574
Oxygen 1	- 0.4856	- 0.4856	- 0.2736
Oxygen 2	0.0974	0.0974	0.0954
Totals	9.2667	9.7743	9.0875

Phase B

With Reference Traverses

(All reactivities in cents)

Axial-1

Material	Sodium Voided Reactivity	Steel Slumped Reactivity	Fuel Slumped Reactivity
304 SSA	- 0.4664	0.0572	0.1139
304 SSB	0.1106	0.1106	0.1145
U-235	0.1507	0.1507	0.1301
Pu-240	0.2191	0.2196	0.1937
U-238	- 3.0748	- 3.0748	- 2.1956
Pu-239	13.046	13.046	10.897
Sodium 1	0.00	0.00	0.00
Sodium 2	0.0002	0.0002	0.0002
Oxygen 1	- 0.4856	- 0.4856	- 0.2732
Oxygen 2	0.0974	0.0974	0.0954
Totals	9.5975	10.122	9.0758

Phase B

With Reference Traverses

(All reactivities in cents)

Axial-2

Material	Sodium Voided Reactivity	Steel Slumped Reactivity	Fuel Slumped Reactivity
304 SSA	- 0.4855	- 0.0210	0.0334
304 SSB	0.1220	0.1220	0.0910
U-235	0.1663	0.1663	0.1486
Pu-240	0.2156	0.2160	0.1918
U-238	- 3.9116	- 3.9116	- 3.1598
Pu-239	13.740	13.740	11.807
Sodium 1	0.00	0.00	0.00
Sodium 2	0.0574	0.0574	0.0574
Oxygen 1	- 0.4856	- 0.4856	- 0.2736
Oxygen 2	0.0974	0.0974	0.0954
Totals	9.5164	9.9813	8.9907

Phase A

(All reactivities in cents)

Axial-1

Material	Sodium Voided with Reference Traverses	Steel Slumped with Sodium Voided Traverses	Fuel Slumped with Steel Slumped Traverses
304 SSA	- 0.7657	0.1422	0.2459
304 SSB	- 0.0762	0.2067	0.1272
U-235	0.1802	0.1802	0.1610
Pu-240	0.1890	0.1894	0.1667
U-238	- 4.1102	- 2.4582	- 1.3479
Pu-239	14.375	12.619	11.428
Sodium 1	0.00	0.00	0.00
Sodium 2	0.0002	0.0002	0.0002
Oxygen 1	- 0.4858	- 0.4858	- 0.2732
Oxygen 2	0.0974	0.0974	0.0954
Totals	9.4035	10.492	10.603

Phase A

(All reactivities in cents)

Axial-2

Material	Sodium Voided with Reference Traverses	Steel Slumped with Sodium Voided Traverses	Fuel Slumped with Steel Slumped Traverses
304 SSA	- 0.7657	- 0.1094	0.0367
304 SSB	- 0.2703	- 0.0163	0.0437
U-235	0.1802	0.1802	0.1610
Pu-240	0.1890	0.1894	0.1667
U-238	- 4.1102	- 3.3763	- 2.4765
Pu-239	14.375	13.709	12.798
Sodium 1	0.00	0.00	0.00
Sodium 2	0.0574	0.05737	0.0574
Oxygen 1	- 0.4856	- 0.4856	- 0.2736
Oxygen 2	0.0974	0.0974	0.0954
Totals	9.2667	10.245	10.609

Phase B

(All reactivities in cents)

Axial-1

Material	Sodium Voided with Reference Traverses	Steel Slumped with Sodium Voided Traverses	Fuel Slumped with Steel Slumped Traverses
304 SSA	- 0.4664	0.1010	0.1330
304 SSB	- 0.1106	0.1650	0.0971
U-235	0.1507	0.1507	0.1301
Pu-240	0.2191	0.2196	0.1937
U-238	- 3.0748	- 2.3482	1.3481
Pu-239	13.046	12.376	10.896
Sodium 1	0.00	0.00	0.00
Sodium 2	0.0002	0.0002	0.0002
Oxygen 1	- 0.4856	- 0.4856	- 0.2732
Oxygen 2	0.0974	0.0974	0.0954
Totals	9.5975	10.276	9.9250

Phase B

(All reactivities in cents)

Axial-2

Material	Sodium Voided with Reference Traverses	Steel Slumped with Sodium Voided Traverses	Fuel Slumped with Steel Slumped Traverses
304 SSA	- 0.4855	- 0.1344	0.0243
304 SSB	0.1220	0.1345	0.1063
U-235	0.1663	0.1663	0.1486
Pu-240	0.2156	0.2160	0.1918
U-238	- 3.9116	- 3.1953	- 2.2244
Pu-239	13.740	13.273	11.193
Sodium 1	0.00	0.00	0.00
Sodium 2	0.0574	0.5737	0.0574
Oxygen 1	- 0.4856	- 0.4856	- 0.2736
Oxygen 2	0.0974	0.0974	0.0954
Totals	9.5164	10.129	9.3187

APPENDIX B

Total Zone Worth Calculated for Each Core Configuration

This appendix lists the data used to calculate the total worth of a zone in the reference and each perturbed configuration. Each zone was divided into radial regions for which the radial traverses of ^{235}Pu were used to determine the radial variation across the zone. Reactivity worth is given for each radial region and the sum of these values is the total zone worth, ρ_D. All reactivities are given in cents using the equation:

$$\rho_z = \sum_i N_i \frac{\rho(r_i)}{\rho(r_4)} \rho_D \qquad \text{(B-1)}$$

where

r_i is the radius of the i^{th} region,

N_i is the number of drawers in the zone in the i^{th} region,

$\rho(r_i)$ is the worth of ^{235}Pu in the 2nd region,

$\rho(r_4)$ is the position where axial traverses were measured, and

ρ_D is the reactivity worth of one drawer in the region determined by r_4.

The term "Large Zone" refers to the size of the region in the core where the sodium voided and steel slumped perturbations were simulated. Small zone refers to the size of the region in the core where the fuel slumped perturbations was simulated.

Phase A

Large Zone

ρ_D = 18.0002 Reference

r_i	N_i	$\rho(r_i)$	$\rho(r_i)/\rho(r_4)$	$N_i[\rho(r_i)/\rho(r_4)]\rho_D$
r_2 = 3.26	8	128.6160	1.0360	149.1857
r_3 = 5.44	16	127.8669	1.0299	296.6145
r_4 = 7.61	24	124.1522	1.0000	432.0048
r_5 = 9.79	28	119.7732	0.9647	486.2142
r_6 = 11.96	28	113.9848	0.9181	462.7275
r_7 = 14.14	16	107.7459	0.8679	249.9580
				Σ = 2076.7047

ρ_D = 19.8092 Sodium Voided

r_2 = 3.26	8	113.4405	1.0236	162.2136
r_3 = 5.44	16	112.7849	1.0176	322.5255
r_4 = 7.61	24	110.8294	1.0000	475.4208
r_5 = 9.79	28	107.7646	0.9723	539.2936
r_6 = 11.96	28	104.0248	0.9386	520.6016
r_7 = 14.14	16	99.8336	0.9008	285.5060
				Σ = 2305.5611

ρ_D = 22.227 Steel Slumped

r_2 = 3.26	8	114.2225	1.0245	182.1725
r_3 = 5.44	16	113.6811	1.0197	362.6380
r_4 = 7.61	24	111.4894	1.0000	533.4480
r_5 = 9.79	28	109.0696	0.9783	608.8509
r_6 = 11.96	28	105.4745	0.9460	588.7488
r_7 = 14.14	16	101.5388	0.9107	323.8741
				Σ = 2599.7321

Phase A

Small Zone

ρ_D = 22.440 Fuel Slumped

r_i	N_i	$\rho(r_i)$	$\rho(r_i)/\rho(r_4)$	$N_i[\rho(r_i)/\rho(r_4)]\rho_D$
r_2 = 3.26	8	91.7722	0.9808	176.0732
r_3 = 5.44	16	92.2862	0.9863	354.1212
r_4 = 7.61	24	93.5714	1.0000	538.5600
r_5 = 9.79	20	95.9701	1.0256	460.2893
r_6 = 11.96	4	99.5092	1.0635	95.4598
				Σ = 1624.5034

ρ_D = 22.227 Steel Slumped

r_2 = 3.26	8	114.2225	1.0245	182.1725
r_3 = 5.44	16	113.6811	1.0197	362.6380
r_4 = 7.61	24	111.4894	1.0000	533.4480
r_5 = 9.79	20	109.0696	0.9783	434.8935
r_6 = 11.96	4	105.4745	0.9460	84.1069
				Σ = 1597.2589

Phase B

Large Zone

ρ_D = 17.8269 Reference

r_i	N_i	$\rho(r_i)$	$\rho(r_i)/\rho(r_4)$	$N_i[\rho(r_i)/\rho(r_4)]\rho_D$
r_2 = 3.26	8	109.1115	0.9514	135.6827
r_3 = 5.44	16	114.0461	0.9944	283.6274
r_4 = 7.61	24	114.6901	1.0000	427.8456
r_5 = 9.79	28	112.1175	0.9776	487.9572
r_6 = 11.96	28	108.4087	0.9452	471.8146
r_7 = 14.14	16	102.5472	0.8941	255.0302
				Σ = 2061.9577

ρ_D = 19.3665 Sodium Voided

r_2 = 3.26	8	102.3269	0.9633	149.2460
r_3 = 5.44	16	105.3599	0.9919	307.3541
r_4 = 7.61	24	106.2246	1.0000	464.7960
r_5 = 9.79	28	103.5667	0.9750	528.7055
r_6 = 11.96	28	99.5943	0.9376	508.4249
r_7 = 14.14	16	95.1465	0.8957	277.5452
				Σ = 2236.0716

ρ_D = 21.1295 Steel Slumped

r_2 = 3.26	8	93.4190	0.9426	159.3333
r_3 = 5.44	16	97.3664	0.9824	332.1219
r_4 = 7.61	24	99.1069	1.0000	507.1080
r_5 = 9.79	28	99.2463	1.0014	592.4543
r_6 = 11.96	28	98.2944	0.9919	586.8338
r_7 = 14.14	16	96.3644	0.9723	328.7074
				Σ = 2506.5588

Phase B

Small Zone

ρ_D = 20.0064 Fuel Slumped

r_i	N_i	$\rho(r_i)$	$\rho(r_i)/\rho(r_4)$	$N_i[\rho(r_i)/\rho(r_4)]\rho_D$
r_2 = 3.26	8	81.2758	0.9550	152.8489
r_3 = 5.44	16	83.1245	0.9767	312.6440
r_4 = 7.61	24	85.1065	1.0000	480.1536
r_5 = 9.79	20	87.1486	1.0240	409.7311
r_6 = 11.96	4	91.1947	1.0715	85.7474
				Σ = 1441.125

ρ_D = 21.1295 Steel Slumped

r_2 = 3.26	8	93.4190	0.9426	159.3333
r_3 = 5.44	16	97.3664	0.9824	332.1219
r_4 = 7.61	24	99.1069	1.0000	507.1080
r_5 = 9.79	20	99.2463	1.0014	423.1816
r_6 = 11.96	4	98.2944	0.9918	83.8250
				Σ = 1505.5698

Phase A

(With Reference Traverses)

Large Zone

ρ_D = 18.6802 Sodium Voided

r_i	N_i	$\rho(r_i)$	$\rho(r_i)/\rho(r_4)$	$N_i[\rho(r_i)/\rho(r_4)]\rho_D$
r_2 = 3.26	8	113.4405	1.0236	152.8865
r_3 = 5.44	16	112.7849	1.0176	303.9807
r_4 = 7.61	24	110.8294	1.0000	448.0848
r_5 = 9.77	28	107.7646	0.9723	508.2850
r_6 = 11.96	28	104.0248	0.9386	490.6678
r_7 = 14.14	16	99.8336	0.9008	269.0899
				Σ = 2172.9947

ρ_D = 19.6854 Steel Slumped

r_i	N_i	$\rho(r_i)$	$\rho(r_i)/\rho(r_4)$	$N_i[\rho(r_i)/\rho(r_4)]\rho_D$
r_2 = 3.26	8	114.2225	1.0245	161.3415
r_3 = 5.44	16	113.6811	1.0197	321.1712
r_4 = 7.61	24	111.4894	1.0000	472.4496
r_5 = 9.77	28	109.0696	0.9783	539.2304
r_6 = 11.96	28	105.4745	0.9460	521.4269
r_7 = 14.14	16	101.5388	0.9107	286.8399
				Σ = 2302.4595

Phase A

Small Zone

ρ_D = 18.1053 Fuel Slumped

r_i	N_i	$\rho(r_i)$	$\rho(r_i)/\rho(r_4)$	$N_i[\rho(r_i)/\rho(r_4)]\rho_D$
r_2 = 3.26	8	91.7722	0.9808	142.0614
r_3 = 5.44	16	92.2862	0.9863	285.7161
r_4 = 7.61	24	93.5714	1.0000	434.5272
r_5 = 9.77	20	95.9701	1.0256	371.3759
r_6 = 11.96	4	99.5092	1.0635	77.0199
				Σ = 1310.7006

ρ_D = 19.6854 Steel Slumped

r_i	N_i	$\rho(r_i)$	$\rho(r_i)/\rho(r_4)$	$N_i[\rho(r_i)/\rho(r_4)]\rho_D$
r_2 = 3.26	8	114.2225	1.0245	161.3415
r_3 = 5.44	16	113.6811	1.0197	321.1712
r_4 = 7.61	24	111.4894	1.0000	472.4496
r_5 = 9.77	20	109.0696	0.9783	385.1645
r_6 = 11.96	4	105.4745	0.9460	74.4896
				Σ = 1414.6165

Phase B

(With Reference Traverses)

Large Zone

ρ_D = 19.1139 Sodium Voided

r_i	N_i	$\rho(r_i)$	$\rho(r_i)/\rho(r_4)$	$N_i[\rho(r_i)/\rho(r_4)]\rho_D$
r_2 = 3.26	8	102.3269	0.9633	147.2994
r_3 = 5.44	16	105.3599	0.9919	303.3452
r_4 = 7.61	24	106.2246	1.0000	458.7336
r_5 = 9.79	28	103.5667	0.9750	521.8095
r_6 = 11.96	28	99.5943	0.9376	501.7934
r_7 = 14.14	16	95.1465	0.8957	273.5452
				Σ = 2206.9062

ρ_D = 20.1033 Steel Slumped

r_2 = 3.26	8	93.4190	0.9426	151.5950
r_3 = 5.44	16	97.3664	0.9824	315.9917
r_4 = 7.61	24	99.1069	1.0000	482.4792
r_5 = 9.79	28	99.2463	1.0014	563.6804
r_6 = 11.96	28	98.2944	0.9919	558.3330
r_7 = 14.14	16	96.3644	0.9723	312.7430
				Σ = 2384.8223

Phase B

Small Zone

ρ_D = 18.0665 Fuel Slumped

r_i	N_i	$\rho(r_i)$	$\rho(r_i)/\rho(r_4)$	$N_i[\rho(r_i)/\rho(r_4)]\rho_D$
r_2 = 3.26	8	81.2758	0.9550	138.0281
r_3 = 5.44	16	83.1245	0.9767	282.3288
r_4 = 7.61	24	85.1065	1.0000	433.5960
r_5 = 9.77	20	87.1486	1.0240	370.0019
r_6 = 11.96	4	91.1947	1.0715	77.4330
				Σ = 1301.3878

ρ_D = 20.1033 Steel Slumped

r_2 = 3.26	8	93.4190	0.9426	151.5950
r_3 = 5.44	16	97.3664	0.9824	315.9917
r_4 = 7.61	24	99.1069	1.0000	482.4792
r_5 = 9.77	20	99.2463	1.0014	402.6289
r_6 = 11.96	4	98.2944	0.9918	79.7538
				Σ = 1432.4486

Phase A

Large Zone

ρ_D = 18.6702 Sodium Voided (with Reference Traverses)

r_i	N_i	$\rho(r_i)$	$\rho(r_i)/\rho(r_4)$	$N_i[\rho(r_i)/\rho(r_4)]\rho_D$
r_2 = 3.26	8	113.4405	1.0236	152.8865
r_3 = 5.44	16	112.7849	1.0176	303.9807
r_4 = 7.61	24	110.8294	1.0000	448.0848
r_5 = 9.79	28	107.7646	0.9723	508.2850
r_6 = 11.96	28	104.0248	0.9386	490.6678
r_7 = 14.14	16	99.8336	0.9008	269.0899
				Σ = 2172.9947

ρ_D = 20.7370 Steel Slumped (with Sodium Voided Traverses)

r_i	N_i	$\rho(r_i)$	$\rho(r_i)/\rho(r_4)$	$N_i[\rho(r_i)/\rho(r_4)]\rho_D$
r_2 = 3.26	8	114.2225	1.0245	169.9605
r_3 = 5.44	16	113.6811	1.0197	338.3283
r_4 = 7.61	24	111.4894	1.0000	497.6880
r_5 = 9.79	28	109.0696	0.9783	568.0362
r_6 = 11.96	28	105.4745	0.9460	549.2817
r_7 = 14.14	16	101.5388	0.9107	302.1630
				Σ = 2424.4576

Phase A

Small Zone

ρ_D = 21.212 Fuel Slumped (with Steel Slumped Traverses)

r_i	N_i	$\rho(r_i)$	$\rho(r_i)/\rho(r_4)$	$N_i[\rho(r_i)/\rho(r_4)]\rho_D$
r_2 = 3.26	8	91.7722	0.9808	166.4378
r_3 = 5.44	16	92.2862	0.9863	334.7423
r_4 = 7.61	24	93.5714	1.0000	509.088
r_5 = 9.77	20	95.9701	1.0256	435.1005
r_6 = 11.96	4	99.5092	1.0635	90.2358
				Σ = 1535.6045

ρ_D = 20.7370 Steel Slumped (with Sodium Voided Traverses)

r_2 = 3.26	8	114.2225	1.0245	169.9605
r_3 = 5.44	16	113.6811	1.0197	338.3283
r_4 = 7.61	24	111.4894	1.0000	497.6880
r_5 = 9.79	20	109.0696	0.9783	405.7401
r_6 = 11.96	4	105.4745	0.9460	78.4688
				Σ = 1490.1857

Phase B

Large Zone

ρ_D = 19.1139 Sodium Voided (with Reference Traverses)

r_i	N_i	$\rho(r_i)$	$\rho(r_i)/\rho(r_4)$	$N_i[\rho(r_i)/\rho(r_4)]\rho_D$
r_2 = 3.26	8	102.3269	0.9633	147.2994
r_3 = 5.44	16	105.3599	0.9919	303.3452
r_4 = 7.61	24	106.2246	1.0000	458.7336
r_5 = 9.79	28	103.5667	0.9750	521.8095
r_6 = 11.96	28	99.5943	0.9376	501.7934
r_7 = 14.14	16	95.1465	0.8957	273.9251
				Σ = 2206.9063

ρ_D = 20.4050 Steel Slumped (with Sodium Voided Traverses)

r_2 = 3.26	8	93.4190	0.9426	153.8701
r_3 = 5.44	16	97.3664	0.9824	320.7339
r_4 = 7.61	24	99.1069	1.0000	489.7200
r_5 = 9.79	28	99.2463	1.0014	572.1398
r_6 = 11.96	28	98.2944	0.9919	566.7122
r_7 = 14.14	16	96.3644	0.9723	317.4365
				Σ = 2420.6125

Phase B

Small Zone

ρ_D = 19.2437 Fuel Slumped (with Steel Slumped Traverses)

	r_i	N_i	$\rho(r_i)$	$\rho(r_i)/\rho(r_4)$	$N_i[\rho(r_i)/\rho(r_4)]\rho_D$
r_2 =	3.26	8	81.2758	0.9550	147.0219
r_3 =	5.44	16	83.1245	0.9767	300.7252
r_4 =	7.61	24	85.1065	1.0000	461.8488
r_5 =	9.77	20	87.1486	1.0240	394.1110
r_6 =	11.96	4	91.1947	1.0715	82.4785
					Σ = 1386.1853

ρ_D = 20.4050 Steel Slumped (with Sodium Voided Traverses)

r_2 =	3.26	8	93.4190	0.9426	153.8701
r_3 =	5.44	16	97.3664	0.9824	320.7339
r_4 =	7.61	24	99.1069	1.0000	489.7200
r_5 =	9.77	20	99.2463	1.0014	408.6713
r_6 =	11.96	4	98.2944	0.9918	80.9507
					Σ = 1453.9461

APPENDIX C

Solution to the Inverse Kinetics Equation

Inverse kinetics techniques are involved either directly or indirectly in many of the experiments and operational measurements at ZPPR. The inverse solution of the point-kinetic equation with an external neutron source is used at ZPPR as the basis for the evaluation of experimental data for small reactivity perturbation measurements. On line computer and data acquisition system make possible the accumulation of necessary data. This appendix is based on Reference 2.

When an external source is present, the point kinetics equation is given by

$$\frac{dn(t)}{dt} = [k(t)(1-\beta)]\ \frac{n(t)}{\ell} + \sum_{j=i}^{m} \lambda_j\ C_j(t) + S \qquad (C\text{-}1)$$

where:

$k(t)$ = system multiplication

β = total effective delayed neutron fraction

ℓ = prompt neutron lifetime

λ_j = decay constant for the jth delayed neutron precursor ($1 \leq j \leq m$)

S = the effective external neutron source

$n(t)$ can be interpreted as the total number of neutrons in the reactor at time t.

The concentration of the delayed neutron precursor for $C_j(t)$ the jth delayed neutron group is given by

$$\frac{d\,C_j(t)}{dt} = \frac{k(t)\;n(t)\;\beta_j}{\ell} - \lambda_i\;C_j(t),\quad (1 \leq j \leq m) \qquad \text{(C-2)}$$

where the delayed neutron fraction of the jth group is defined as β_j, and

$$\beta = \sum_{j=i}^{m} \beta_j$$

Solving for k(t) in terms of the power history, Equation C-1 can be rewritten

$$\frac{dn(t)}{dt} = k(t)(1-\beta)\;\frac{n(t)}{\ell} - \frac{n(t)}{\ell} + \sum_{j=1}^{m} \lambda_j\;C_j(t) + S$$

$$\frac{dn(t)}{dt} + \frac{n(t)}{\ell} - \sum_{j=1}^{m} \lambda_j\;C_j(t) + S = k(t)(1-\beta)\;\frac{n(t)}{\ell}$$

$$k(t) = \frac{dn(t)}{dt}\;\frac{\ell}{(1-\beta)\;n(t)} + \frac{n(t)\;\ell}{\ell\;(1-\beta)\;n(t)}$$

$$- \sum_{j=1}^{m} \lambda_j\;C_j(t)\;\frac{\ell}{(1-\beta)\;n(t)} + \frac{S\;\ell}{(1-\beta)\;n(t)}$$

$$k(t) = \frac{1}{n(t)}\;\frac{dn(t)}{dt}\;\frac{\ell}{(1-\beta)} + \frac{1}{(1-\beta)} - \sum_{j=1}^{m} \lambda_j\;C_j(t)\;\frac{\ell}{(1-\beta)\;n(t)}$$

If two new variables $A_j(t)$ and $\alpha(t)$ are defined such that

$$A_j(t) = \frac{\ell\ \lambda_j C_j(t)}{n(t)} \quad \text{and} \quad \alpha(t) = \frac{1}{n(t)} \frac{dn(t)}{dt} \tag{C-3}$$

$$k(t) = \frac{\alpha(t) + 1}{(1-\beta)} - \frac{1}{(1-\beta)} \sum_{j=1}^{m} A_j(t) - \frac{S\ \ell}{(1-\beta)\ n(t)} \tag{C-4}$$

and Equation C-2

$$\frac{d\ C_j(t)}{dt} = \frac{k(t)\ n(t)\ \beta_i}{\ell} - \lambda_j\ C_j(t), \quad (1 \leq j \leq m)$$

$$\frac{\ell\ dC_j(t)}{dt} = \ell\ \lambda_j\ C_j(t) = k(t)\ n(t)\ \beta_j, \quad (1 \leq j \leq m)$$

$$\frac{\ell\ dC_j(t)}{dt} = k(t)\ n(t)\ \beta_j - \ell\ \lambda_j\ C_i(t), \quad (1 \leq j \leq m)$$

but

$$A_j(t) = \frac{\ell\ \lambda_i C_i(t)}{n(t)} \quad \text{then} \quad C_i(t) = \frac{A_j(t)\ n(t)}{\ell\ \lambda_i}$$

and

$$\frac{\ell}{\ell\ \lambda_i}\ A_j(t)\ \frac{d\ n(t)}{dt} + n(t)\ \frac{dA_j(t)}{dt} = k(t)\ n(t)\ \beta_j$$

$$- \frac{\ell\ \lambda_i\ A_j(t)\ n(t)}{\ell\ \lambda_i}$$

$$\frac{n(t)}{\lambda_i}\frac{dA_j(t)}{dt} = k(t)\, n(t)\, \beta_j - A_j(t)\, n(t) - \frac{A_j(t)}{\lambda_i}\frac{dn(t)}{dt}$$

$$\frac{dA_j(t)}{dt} = \frac{k(t)\, n(t)\, \beta_j\, \lambda_i}{n(t)} - \frac{A_j(t)\, n(t)\, \lambda_i}{n(t)}$$

$$- \frac{A_j(t)}{\lambda_i}\frac{\lambda_i}{n(t)}\frac{dn(t)}{dt}$$

$$\frac{dA_j(t)}{dt} = k(t)\, \beta_j\, \lambda_i - A_j(t)\, \lambda_i - A_j(t)\frac{1}{n(t)}\frac{dn(t)}{dt}$$

from above

$$\alpha(t) = \frac{1}{n(t)}\frac{dn(t)}{dt}$$

$$\frac{dA_j(t)}{dt} = k(t)\, \beta_j\, \lambda_i - \lambda_j + \alpha(t)\ A_j(t), \quad (1 \leq j \leq m) \qquad \text{(C-5)}$$

The variable $\alpha(t)$ is defined as the instantaneous inverse period. The solution to the inhomogeneous differential Equation (C-5) is

$$A_j(t) = A_j(0)e^{-[\lambda_j - \alpha(t)]t} + \int_0^t e^{-[\lambda_j + \alpha(t)]} k(s)\beta_j\lambda_j\, ds \qquad \text{(C-6)}$$

Using integration by parts to solve the integral in Equation (C-6) for discrete time steps, and substituting the result in Equation (C-4) to solve for k(t), time step by time step the result is

$$\delta_i = \frac{\ell(\alpha_i - \alpha_{i-1}) + S\ell\left(\frac{1}{\phi_{i-1}} - \frac{1}{\phi_i}\right) - \sum_{j=1}^{m} \frac{\lambda_j \beta_j k_{j-1} E_{i,j}}{(\lambda_i + \alpha_i)} - A_{j,i-1} E_{i,j}}{1 - \beta + \sum_{j=1}^{m} \frac{\lambda_j \beta_j}{(\lambda_j + \alpha_i)} 1 - \frac{E_{i,j}}{\Delta t(\lambda_j + \alpha_i)}} \qquad \text{(C-7)}$$

The subscript i denotes the ith time step,

$$E_{i,j} = 1 - e^{-(\lambda_j + \alpha_i)\Delta t}$$

and

δ_i = change in k during time step i

so

$$k_i = k_{i-1} + \delta_i$$

The precursor concentrations at time step i are given by

$$A_{j,i} = A_{j,i-1} e^{-(\lambda_j + \alpha_i)\Delta t} + \frac{\beta_j \lambda_j}{(\lambda_j + \alpha_i)} k_{i-1} E_{i,j} + \delta_i 1 - \frac{E_{i,j}}{\Delta t(\lambda_j + \alpha_i)} \qquad \text{(C-8)}$$

The assumption is made that the reactivity is a linear function over a time interval and that the power (count rate) behaves exponentially over the time interval.

Equations (C-7) and (C-8) form the basis for the algorithms used at ZPPR.

The initial conditions are that the reactor has been at a constant power for a sufficiently long time and that all delayed neutron precursor concentrations have reached equilibrium.

APPENDIX D

Coefficients of the Polynomial Curve Fits to Reactivity Worth Profile Data

This appendix lists the coefficients of the least square polynomial curve fits to the reactivity worth profile data. The information is given for the reference and each perturbed configuration for Phase A halves 1 and 2, and Phase B halves 1 and 2. The coefficients are listed for each small sample identification (see Table 1) and the range, in inches, over which the curve was fitted.

Four significant figures are presented in lieu of the eight place output given by the computer analysis of measured data. The data listed are input to the computer code that calculates the reactivity worth of a drawer in ZPPR.

Phase A

Half 1

Reference Configuration

Pu-240

0 to 22 inches

A_o	13.9546
A_1	- 0.8269
A_2	0.1027
A_3	- 0.0082
A_4	0.0002

Pu-30

0 to 22 inches

A_o	124.9395
A_1	0.1888
A_2	- 0.3806
A_3	- 0.0982
A_4	0.0124
A_5	- 0.00002

U-6

0 to 36 inches

A_o	86.6589
A_1	1.9268
A_2	- 0.6923
A_3	- 0.0329
A_4	- 0.0006
A_5	0.00001

Du-6

0 to 36 inches

A_o	- 5.9718
A_1	- 0.2270
A_2	0.7056
A_3	- 0.0030
A_4	0.00004

Phase A

Half 1

Reference Configuration (continued)

SS-1

0 to 18 inches

Part 1

A_o	- 5.6638
A_1	0.5632
A_2	- 0.6932
A_3	- 0.0063
A_4	- 0.0002

SS-1

18 to 37.57 inches

Part 2

A_o	-73.8250
A_1	15.2246
A_2	- 1.2102
A_3	0.0469
A_4	0.0009
A_5	0.00001

Sodium Voided Configuration

Pu-30

0 to 36 inches

A_o	111.8862
A_1	0.1499
A_2	- 0.4794
A_3	- 0.1239

Du-6

0 to 36 inches

A_o	- 4.2195
A_1	- 0.3614
A_2	- 0.0968
A_3	- 0.0052
A_4	0.0001
A_5	- 0.000001

Phase A

Half 1

Sodium Voided Configuration (continued)

SS-1		Part 1
0 to 18 inches	A_o	- 3.4207
	A_1	- 0.0199
	A_2	0.0313
	A_3	- 0.0009
SS-1		Part 2
18 to 37.59 inches	A_o	- 6.4550
	A_1	2.0166
	A_2	- 0.1456
	A_3	0.0040
	A_4	- 0.00004

Steel Slumped Configuration

Pu-30		
0 to 22 inches	A_o	111.3787
	A_1	0.7256
	A_2	- 0.6621
	A_3	- 0.0537
	A_4	- 0.0030
	A_5	0.0001
Du-6		
0 to 36 inches	A_o	- 3.8588
	A_1	- 0.4093
	A_2	0.1063
	A_3	- 0.0060
	A_4	0.0001
	A_5	- 0.000001

Phase A

Half 1

Steel Slumped Configuration (continued)

SS-1		Part 1
0 to 18 inches	A_o	- 3.5602
	A_1	- 0.7270
	A_2	0.2857
	A_3	- 0.0358
	A_4	0.0021
	A_5	- 0.0001

SS-1		Part 2
18 to 36 inches	A_o	45.4952
	A_1	- 5.8699
	A_2	0.2911
	A_3	- 0.0065
	A_4	0.0001

Fuel Slumped Configuration

Pu-240		
0 to 22 inches	A_o	14.8808
	A_1	- 3.2618
	A_2	0.8490
	A_3	- 0.0755
	A_4	0.0026
	A_5	- 0.00003

Pu-30		
0 to 22 inches	A_o	110.8400
	A_1	-17.7140
	A_2	5.1261
	A_3	- 0.5502
	A_4	0.0231
	A_5	- 0.0003

Phase A

Half 1

Fuel Slumped Configuration (continued)

U-6

0 to 36 inches		
	A_o	-33.2663
	A_1	33.2038
	A_2	- 3.7506
	A_3	0.1709
	A_4	- 0.0036
	A_5	0.00003

Du-6

0 to 36 inches		
	A_o	- 3.3876
	A_1	- 0.8436
	A_2	0.1572
	A_3	- 0.0083
	A_4	0.0002
	A_5	- 0.000002

SS-1 Part 1

0 to 18 inches		
	A_o	- 4.8891
	A_1	2.3935
	A_2	- 0.6138
	A_3	0.0533
	A_4	- 0.0014

SS-1 Part 2

18 to 36 inches		
	A_o	- 0.0034
	A_1	6.1941
	A_2	- 0.3816
	A_3	0.0099
	A_4	- 0.00009

Phase A

Half 2

Sodium Voided Configuration

Pu-30

0 to 22 inches	A_o	110.5948
	A_1	1.5862
	A_2	- 0.5421
	A_3	0.0148
	A_4	- 0.0001

Du-6

0 to 36 inches	A_o	- 4.5411
	A_1	- 0.2899
	A_2	0.0656
	A_3	- 0.0027
	A_4	0.00003

SS-1 Part 1

0 to 20 inches	A_o	- 3.4445
	A_1	- 0.1689
	A_2	0.0454
	A_3	- 0.0013

SS-1 Part 2

20 to 37.57 inches	A_o	6.8421
	A_1	- 0.3939
	A_2	0.0056

Phase A

Half 2

Steel Slumped Configuration

Pu-30

0 to 22 inches	A_o	114.4753
	A_1	- 1.1246
	A_2	0.0411
	A_3	- 0.0238
	A_4	0.0007

Du-6

0 to 37.57 inches	A_o	- 3.9577
	A_1	- 0.2963
	A_2	0.0626
	A_3	- 0.0026
	A_4	0.00003

SS-1 Part 1

0 to 18 inches	A_o	- 4.8819
	A_1	0.4522
	A_2	- 0.0786
	A_3	0.0089
	A_4	- 0.0003

SS-2 Part 2

18 to 36 inches	A_o	69.0294
	A_1	-10.4375
	A_2	0.5965
	A_3	- 0.0151
	A_4	0.0001

Phase A

Half 2

Fuel Slumped Configuration

Pu-240

0 to 22 inches	A_o	14.9040
	A_1	- 3.3616
	A_2	1.0244
	A_3	- 0.0959
	A_4	0.0035
	A_5	- 0.00004

Pu-30

0 to 22 inches	A_o	112.4485
	A_1	-18.3854
	A_2	5.2712
	A_3	- 0.5449
	A_4	0.0222
	A_5	- 0.0003

U-6

0 to 36 inches	A_o	-24.2420
	A_1	30.0202
	A_2	- 3.2822
	A_3	- 0.1466
	A_4	- 0.0030
	A_5	0.00002

Du-6

0 to 22 inches	A_o	- 4.1700
	A_1	- 0.4301
	A_2	0.0778
	A_3	- 0.0031
	A_4	0.00004

Phase A

Half 2

Fuel Slumped Configuration (continued)

SS-1 — Part 1

0 to 18 inches

A_o	- 2.9761
A_1	0.1732
A_2	0.0611
A_3	- 0.0342
A_4	0.0035
A_5	- 0.0001

SS-1 — Part 2

18 to 36 inches

A_o	5.5067
A_1	- 0.3365
A_2	0.0051

Phase B

Half 1

Reference Configuration

Pu-240

0 to 22 inches

A_o	14.0268
A_1	0.3382
A_2	- 0.0798
A_3	0.0018

Pu-30

0 to 22 inches

A_o	118.1895
A_1	0.4661
A_2	- 0.5875
A_3	0.0160

U-6

0 to 36 inches

A_o	80.5703
A_1	1.1433
A_2	- 0.5890
A_3	0.0264
A_4	- 0.0004
A_5	0.000002

Du-6

0 to 36 inches

A_o	- 4.8121
A_1	- 0.4097
A_2	0.1090
A_3	- 0.0061
A_4	0.0001
A_5	- 0.000001

Phase B

Half 1

Reference Configuration (continued)

SS-1		Part 1
0 to 18 inches	A_o	- 4.2727
	A_1	0.2901
	A_2	- 0.0277
	A_3	0.00407
	A_4	- 0.0001

SS-1		Part 2
18 to 36 inches	A_o	50.9745
	A_1	- 6.9463
	A_2	0.3590
	A_3	- 0.0082
	A_4	0.0001

Sodium Voided Configuration

Pu-30		
0 to 22 inches	A_o	108.6161
	A_1	0.3672
	A_2	- 0.4816
	A_3	0.0124

Du-6		
0 to 37.57 inches	A_o	- 3.9145
	A_1	- 0.3356
	A_2	- 0.0837
	A_3	- 0.0041
	A_4	0.00007
	A_5	- 0.0000003

Phase B

Half 1

Sodium Voided Configuration (continued)

SS-1 — Part 1

0 to 18 inches

A_o	- 4.2970
A_1	0.3736
A_2	- 0.0559
A_3	- 0.0064
A_4	- 0.0002

SS-1 — Part 2

18 to 37.57 inches

A_o	9.7202
A_1	- 0.5951
A_2	- 0.0092

Steel Slumped Configuration

Du-6

0 to 37.57 inches

A_o	- 3.6926
A_1	- 0.1473
A_2	0.0569
A_3	- 0.0029
A_4	0.00005
A_5	- 0.0000002

SS-1 — Part 1

0 to 18 inches

A_o	- 2.9128
A_1	- 0.6901
A_2	0.2332
A_3	- 0.0282
A_4	0.0017
A_5	- 0.00004

Phase B

Half 1

Steel Slumped Configuration (continued)

SS-1

18 to 37.57 inches

	Part 2
A_o	- 5.4253
A_1	1.7129
A_2	- 0.1234
A_3	0.0033
A_4	- 0.00003

Fuel Slumped Configuration

Pu-240

0 to 22 inches

A_o	13.6002
A_1	- 0.0186
A_2	0.4461
A_3	- 0.0337
A_4	0.0007

Pu-30

0 to 22 inches

A_o	101.4728
A_1	-16.6993
A_2	4.7683
A_3	- 0.5123
A_4	0.0217
A_5	- 0.0003

U-6

0 to 36 inches

A_o	52.0308
A_1	2.8944
A_2	- 0.2579
A_3	- 0.0102
A_4	0.0008
A_5	- 0.00001

Phase B

Half 1

Fuel Slumped Configuration (continued)

Du-6

0 to 36 inches	A_o	- 2.6514
	A_1	- 0.7857
	A_2	0.1363
	A_3	- 0.0069
	A_4	0.0001
	A_5	- 0.000001

SS-1 Part 1

0 to 18 inches	A_o	- 2.0692
	A_1	- 0.4718
	A_2	0.1554
	A_3	- 0.0314
	A_4	0.0027
	A_5	- 0.0001

SS-1 Part 2

18 to 37.57 inches	A_o	0.0779
	A_1	- 0.4848
	A_2	0.0075

Phase B

Half 2

Reference Configuration

Pu-240

0 to 22 inches

A_o	13.9105
A_1	0.1987
A_2	- 0.0576
A_3	0.0009

Pu-30

0 to 22 inches

A_o	117.5793
A_1	1.3472
A_2	- 0.6133
A_3	- 0.0159

U-6

0 to 36 inches

A_o	79.5505
A_1	2.1822
A_2	- 0.6810
A_3	0.0312
A_4	- 0.0006
A_5	0.000004

Du-6

0 to 36 inches

A_o	- 4.9374
A_1	- 0.4642
A_2	0.1102
A_3	- 0.0060
A_4	- 0.0001
A_5	- 0.000001

Phase B

Half 2

Reference Configuration (continued)

SS-1		Part 1
0 to 18 inches	A_o	- 3.2140
	A_1	- 0.3935
	A_2	0.1043
	A_3	- 0.0058
	A_4	0.0001
SS-1		Part 2
18 to 37.57 inches	A_o	4.5525
	A_1	- 0.2349
	A_2	0.0030

Sodium Voided Configuration

Pu-30		
0 to 22 inches	A_o	108.8698
	A_1	1.0991
	A_2	- 0.4778
	A_3	- 0.0113
Du-6		
0 to 36 inches	A_o	- 4.2495
	A_1	- 0.2407
	A_2	0.0570
	A_3	- 0.0023
	A_4	0.00003
SS-1		Part 1
0 to 18 inches	A_o	- 3.8382
	A_1	0.0424
	A_2	0.136

Phase B

Half 2

Sodium Voided Configuration (continued)

SS-1

18 to 37.57 inches

Part 2

A_o - 8.5075
A_1 1.8414
A_2 - 0.1178
A_3 0.0030
A_4 - 0.00003

Steel Slumped Configuration

Pu-30

0 to 22 inches

A_o 98.9377
A_1 1.1672
A_2 - 0.5161
A_3 0.0357
A_4 - 0.0020
A_5 0.00004

Du-6

0 to 36 inches

A_o - 3.8390
A_1 - 0.1592
A_2 0.0451
A_3 - 0.0018
A_4 0.00002

SS-1

Part 1

0 to 18 inches

A_o - 3.2914
A_1 - 0.1853
A_2 0.0486
A_3 - 0.0013

Phase B

Half 2

Fuel Slumped Configuration (continued)

Du-6

0 to 36 inches

A_o	- 3.1267
A_1	- 0.4730
A_2	0.0756
A_3	- 0.0029
A_4	0.00004

SS-1 Part 1

0 to 18 inches

A_o	- 4.8254
A_1	2.2227
A_2	- 0.5691
A_3	0.0485
A_4	- 0.0013

SS-1 Part 2

18 to 36 inches

A_o	3.4151
A_1	- 0.1740
A_2	0.0022

Phase B

Half 2

Steel Slumped Configuration (continued)

		Part 2
SS-1		
18 to 37.57 inches	A_o	-54.8022
	A_1	9.1491
	A_2	- 0.5414
	A_3	0.0138
	A_4	- 0.0001

Fuel Slumped Configuration

Pu-240		
0 to 22 inches	A_o	10.2045
	A_1	- 0.6478
	A_2	0.3142
	A_3	0.0278
	A_4	0.0006
Pu-30		
0 to 22 inches	A_o	101.4685
	A_1	-16.2026
	A_2	4.7004
	A_3	- 0.4972
	A_4	0.0207
	A_5	- 0.0003
U-6		
0 to 36 inches	A_o	53.0668
	A_1	2.3492
	A_2	- 0.1029
	A_3	- 0.0178
	A_4	0.0009
	A_5	- 0.00001

APPENDIX E

Computer Program to Calculate ZPPR Drawer Reactivity Worths from Small-Sample Traverse Data

I. Input Instructions

First Card Title

Second Card 12(I6)

NMAT - number of materials (10 maximum)
NINC - total number of increments in a drawer
NSTEEL - material number for steel--if more than one steel material is used, NSTEEL must be negative and numerically equal to the number of steel used.

* (multiple steels must be the first material read)

NPDOP - non-zero value eliminates pointwise data printout.

Third Card 12(I6)

NPY(I) - order of polynomial fit for I^{th} material. Negative value indicates range of validity less than 1-NINC as in the case of multiple steels (a number for each material)

Fourth Card 6(E12.6)

SHT - shim height in inches
BETA - effective beta for reactor
DKIH - inhours per percent delta k

Fifth Card 20(A4) in program; 10(A8) in cards

MNAM - material name (8 characters maximum)

Sixth and Seventh Cards 6(E12.4)

(FACT(I), I=1, NMAT)
FACT - constant multiplier for a material worth- default = 1.0

Eighth Card 12(I6)

NSM - number of shims at each location
NLC - number of shim locations
NBS(I) - increment number (in unshimmed drawer) corresponding to the first shim of the Ith group of shims, ordered from the front of the drawer. NBS(I=1,6)

For each material 9, 10 (if needed), 11

Ninth Card 4(E12.4,I6) 72 MNAM

AM(I) - mass of material, in kg, over the Ith interval
IL(I) - number of last increment in the Ith interval
AM(I), IL(I), I = 1,4

Tenth Card 12(I6)

LMIN
LMAX - range of increments where the Ith material applies, to be used only when NPY(I) is negative (which indicates range of validity less than 1-NINC, as in the case of multiple steels.

Eleventh Card (6E12.5, I8)

AØ
A(I),I=1,5 - coefficients of polynomial fit to reactivity worth profile data, polynomial terms of form $A(N)*X^{(N-1)}$, reactivity in IH/kg.
MATNØ

A pair of cards
1) AM and IL values
2) A(I) values . . .

should be included for each material in the correct order. Each A(I) card must have the material number right adjusted in columns 72-80.

II. Program Listing

```
C
      DIMENSION AMASS(720),BMASS(720),RHO(720),A(6),
     1  RRHO(10),EXRHO(10),AM(4),MNAM(22),NBS(6),NPY(10),
     2  IL(4),NITLE(20),FACT(10)
      INTEGER T1,T2
      DATA T1,T2/4HTOTA,4HLS /
C          PROGRAM TO CALCULATE ZPPR DRAWER WORTHS FROM
C          SMALL-SAMPLE TRAVERSES DATA, R. E. KAISER,
C          JANUARY 1975
C
  500 READ (4,8003) (NITLE(I),I=1,20)
      READ (4,8000)NMAT,NINC,NSTEEL,NPDOP
      IF(NMAT,EG.0) CALL EXIT
      READ (4,8000) (NPY(I),I=1,NMAT)
 8000 FORMAT (12I6)
      READ (4,8002) SHT,BETA,DKIH
 8002 FORMAT (6E12.6)
      SHIM=SHT*0.015710
C
C           INPUT PARAMETERS
C       NITLE - PROBLEM TITLE, UP TO 80 CHARACTERS
C       NMAT - NO. OF MATERIALS, 10 MAXIMUM
C       NINC - TOTAL NO. OF INCREMENTS IN DRAWER
C       NSTEEL - MATERIAL NUMBER FOR STEEL
C          IF MORE THAN ONE STEEL MATERIAL IS USED,
C          NSTEEL MUST BE NEGATIVE AND NUMERICALLY
C          EQUAL TO THE NUMBER OF STEELS USED
C        **NOTE**MULTIPLE STEELS MUST BE THE FIRST
C            MATERIALS USED
C       NPDOP - NON ZERO VALUE ELIMINATES POINTWISE DATA
C          PRINTOUT
C       NPY(I) = ORDER OF POLYNOMIAL FIT FOR ITH MATERIAL
C          NEGATIVE VALUE INDICATES RANGE OF VALIDITY
C          LESS THAN 1-NINC, AS IN THE CASE OF MULTIPLE
C          STEELS
C       SHT - SHIM HEIGHT IN INCHES
C       BETA - EFFECTIVE BETA FOR REACTOR
C       DKIH - INHOURS PER PERCENT DELTA K
C       MNAM - MATERIAL NAME (8 CHARACTER MAXIMUM)
C       FACT - CONSTANT MULTIPLIER FOR A MATERIAL WORTH -
C          DEFAULT=1.0
C       NSM - NUMBER OF SHIMS AT EACH LOCATION
C       NLC - NUMBER OF SHIM LOCATIONS
C       NBS(I) - INCREMENT NUMBER (IN UNSHIMMED DRAWER)
C          CORRESPONDING TO FIRST SHIM OF ITH GROUP OF
C          SHIMS, ORDERED FROM FRONT OF DRAWER
C     * AM(I) - MASS OF MATERIAL, IN KG, OVER ITH INTERVAL
C     * IL(I) - NUMBER OF LAST INCREMENT IN ITH INTERVAL
```

```
C     * LMIN,LMAX - RANGE OF INCREMENTS WHERE ITH
C          MATERIAL APPLIES.  TO BE USED ONLY WHEN NPY(I)
C          IS NEGATIVE
C     * A(I) - COEFFICIENTS OF POLYNOMIAL FIT TO
C          REACTIVITY DATA POLYNOMIAL TERMS OF FORM
C          A(N)*(X**(N-1))
C          REACTIVITY IN IH/KG
C
C     * A PAIR OF CARDS... 1 - AM and IL VALUES
C                          2 - A(I) VALUES
C       SHOULD BE INCLUDED FOR EACH MATERIAL IN THE
C       CORRECT ORDER, EACH A(I) CARD MUST HAVE THE
C       MATERIAL NUMBER, RIGHT ADJUSTED, IN COLUMNS
C       72-80
C
      IM=2*NMAT
      READ (4,8003) (MNAM(I), I=1,IM))
      MNAM(IM+1) = T1
      MNAM(IM+2) = T2
      READ (4,8002) (FACT(I), I=1,NMAT))
 8003 FORMAT (20A4)
      READ (4,8000) NSM,NLC,(NBS(I),I=1,6))
      WRITE (5,9013) (NITLE(I),I=1,20))
 9013 FORMAT (1H1,20A4//)
      WRITE (5,9005) NMAT,NINC,NSM,NLC,SHT,BETA,DKIH
 9005 FORMAT (1H,20X,18HNO, OF MATERIALS = I3/21X,18HNO,
     1OF INTERVALS = I3/21X,20SHIMS PER LOCATION = ,
     2I3/21X,16HSHIM LOCATIONS = ,I3/21X,I3HSHIM HEIGHT=
     3,F7,3/21X,16HBETA EFFECTIVE = ,F10,7/21X,21HINHOURS
     4PER PERCENT = ,F8,2///)
      WRITE (5,9007)
 9007 FORMAT (1H,20X,24HTERMS IN POLYNOMIAL FITS/)
      DO 3 KJ=1,NMAT
      K1=2+KJ=1
      K2=K1+1
    3 WRITE (5,9008) MNAM(K1),MNAM(K2),NPY(KJ)
 9008 FORMAT (1H,24X,2A4,I8)
      NTS=NLC*NSM
      DO 100 JMAT=1,NMAT
      CFAC=FACT(JMAT)
      IF(CFAC.EQ.0.0) CFAC=1.0
      LMIN=1
      LMAX=NINC
      NTM=NPY(JMAT)
      NTM=IABS(NTM)
      NEXT=0
      LU=0
    2 READ (4,8001) (AM(I),IL(I),I=1,4))
 8001 FORMAT (4(E12.4,I6))
      DO 5 I=1,4
      LL=LU+1
      LU=IL(I)
```

```
      IF(LU.EQ.NINC) NEXT=1
      DO 10 L=LL,LU
   10 AMASS(L)=AM(I)
      IF(NEXT,EQ.1) GO TO 20
    5 CONTINUE
      GO TO 2
C
C          SHIM ADJUSTMENT
C
   20 NU=NINC
      IF(NPY(JMAT),LT.0) READ (4,8000) LMIN,LMAX
   22 DO 41 I=1,5
      IF(NBS(I+1),EQ.0) GO TO 42
   41 CONTINUE
   42 ISLC=I
   48 NL=NBS(ISLC)+NSM+ISLC
   50 DO 55 L=NL,NU
      LD=NSM*ISLC
      Lx1=L=LD
   55 BMASS(L)=AMASS(Lx1)
      IF(NL.EQ.1) GO TO 70
      ISLC=ISLC=1
      NU=NL=NSM-1
      SHM=0.0
      IF((NSTEEL.EQ.JMAT),OR,(NSTEEL.LT.0)) SHM=SHIM
   59 DO 60 L=1,NSM
      Lx2=NL=L
      IF((Lx2.GT.LMAX),OR,(Lx2.LT.LMIN)) SHM=0,0
   60 BMASS(Lx2)=SHM
      IF(ISLC,FT,0) GO TO 48
      IF((NBS(1).EQ.1,OR,(NU.LT.0)) GO TO 70
      NL=1
      GO TO 50
   70 READ (4,8004) AD,(A(I),I=1,5),MATNO
 8004 FORMAT (6E12,5,I8)
      IF(JMAT,NE,MATNO) GO TO 7000
      XLMIN=LMIN
      X=0.0625*XLMIN=0.03125
      JM=JMAT
      DO 72 N=1,NINC
   72 RHO(N)=0.0
      RRHO(JM)=0.0
      EXRHO(JM)=0.0
      DO 80 I=LMIN,LMAX
      RHO(I)=A(NTM)*X
      DO 75 N=2,NTM
      Lx3=NTM=N+1
   75 RHO(I)=(RHO(I)+A(Lx3)*X
      RHO(I)=(RHO(I)+AO)/(BETA+DKIH)*CFAC
      RRHO(JM)=RRHO(JM)+RHO(I)*AMASS(I)
      EXRHO(JM)=EXRHO(JM)+RHO(I)*BMASS(I)
```

```
   80 X=X+0.0625
      IF(NPDOP,NE.0) GO TO 99
      JMX=2*JMAT-1
      JMY=JMX+1
      WRITE (5,9006) MNAM(JMX),MNAM(JMY)
 9006 FORMAT (21H1 POINTWISE DATA FOR ,2A4//)
      WRITE (5,9009)
 9009 FORMAT (1H,10X,23HPOLYNOMIAL COEFFICIENTS)
      KX=0
      WRITE (5,9010) KX,A0
 9010 FORMAT (1H,I15,E16,5)
      NTZ=NTM-1
      WRITE (5,9011) (KX,A(KX),KX=1,NTZ)
 9011 FORMAT (1H,I15,E16,5,/4(I16,E16,5/))
      WRITE (5,9012) LMIN,LMAX
 9012 FORMAT (1H,//16X,34HREACTIVITIES SUMMED OVER
     1INTERVALS,I4,8H TRHOUGH,I4//)
      WRITE (5,9000)
 9000 FORMAT (1H,18X,9HREFERENCE,3X,8HEXPANDED,20X,
     19HREFERENCE,3X,8HEXPANDED,20X,9HREFERENCE,3X,
     28HEXPANDED/3X,1HI,2X,10HRHO,C/KGM,4X,6HMASSES,
     36X,6HMASSES,5X,1HI,2X,10HRHO,C/KGM,4X,6HMASSES,
     46X,6HMASSES,5X,1HI,2X,10HRHO,C/KGM,4X,6HMASSES,
     56X,6HMASSES//)
      DO 90 I=LMIN,LMAX,3
      I2=I+1
      I3=I+2
   90 WRITE (5,9001) I,RHO(I),AMASS(I),BMASS(I),I2,RHO
     1(I2),AMASS(I2),BMASS(I2),I3,RHO(I3),AMASS(I3),
     2BMASS(I3)
 9001 FORMAT (3(I4,3E12,5))
   99 IF(NSTEEL,LT,O) NSTEEL=NSTEEL+1
  100 CONTINUE
      WRITE (5,9013) (NITLE(I),I=1,20)
      WRITE (5,9002)NSM,NLC
 9002 FORMAT (26H EXPANSION REACTIVITY FOR I2,6H SHIMS/
     112H AT EACH 1I2,10H POSITIONS//27H ALL REACTIVITIES
     2IN CENTS///)
      WRITE (5,9003)
 9003 FORMAT (16H MATERIAL,16H REFERENCE,16H EXPANDED,
     1,16H DIFFERENTIAL /16X,16H REACTIVITY,16H
     2REACTIVITY,213H REACTIVITY//)
      TRR=0.0
      TEX=0.0
      TDF=0.0
      DO 110 I=1,NMAT
      DIF=EXRHO(I)=RRHO(I)
      TDF=TDF+DIF
      TRR=TRR+RRHO(I)
      TEX=TEX+EXRHO(I)
      L=2*I=1
```

```
 110 WRITE (5,9004) MNAM(L),MNAM(L+1),RRHO(I),
    1EXRHO(I),DIF
9004 FORMAT (1H,3X,2A4,5X,1PE12,4,4X,E12,4,4,4X,E12,4)
     L=2*NMAT+1
     WRITE (5,9004) MNAM(L),MNAM(L+1),TRR,TEX,TDF
     GO TO 500
7000 WRITE (5,7001) MATNO,JMAT
7001 FORMAT (37H ERROR IN REACTIVITY INPUT, COEF CARD,
    1I2,8H IN POS.,I2)
     CALL EXIT
     END
```

III. Sample Problem Input Case I

```
ZPPR-5  PHASE A  REFERENCE  MASTER  05-1-101
    10   576     -2     1
    -5    -6      6     5     5     6     -5     -4     -5     -4
 .776      0.0031248       1030.08
 304 SSA 304 SSB   U-235   Pu-240   U-238   Pu-239 SODIUM1 SODIUM2 OXYGEN1 OXYGEN2
 1.0         1.0          1.0685      0.9831       1.0333      1.0282
 1.0         1.0          1.0         1.0
     0     0     0
  3.416 E-03   288  2.065 E-03    512  6.929 E-03    576
     1   288
 -.56638E 01  .56320     -.69321E-01   .62856E-02  -.17267E-03                    1
  3.416 E-03   288  2.065 E-03    512  6.929 E-03    576
   289   576
 -.73825E 02  .15225E 02 -.12102E 01   .46915E-01  -.89432E-03   .67313E-05       2
  2.398 E-05   288  3.038 E-05    512  0.0          576
  .86659E 02  .19268E 01 -.69235       .32865E-01  -.63819E-03   .46582E-05       3
  2.262 E-04   288  0.0           576
  .13955E 02 -.82689      .10270      -.82339E-02   .18605E-03                    4
  1.116 E-02   288  1.416 E-02    512  0.0          576
 -.59718E 01 -.22697      .70563E-01  -.29904E-02   .37076E-04                    5
  1.727 E-03   288  0.0           576
  .12494E 03  .18882     -.38058      -.98160E-02   .12440E-02   -.21475E-04      6
  1.717 E-03   288  1.295 E-03    512  2.076 E-03    576
     1   272
-0.60308E 01-0.27176    0.14310      -0.1458E-01  0.86712E-03   0.20683E-04      7
  1.717 E-03   288  1.295 E-03    512  2.076 E-0     576
   273   576
  0.59634E 02-0.72497E 01 o.33290    -0.68368E-02  0.52934E-04                    8
   1.772 E-03  288  2.039 E-03    512  1.031 E-07    576
     1   272
-0.98429E 01-0.44102     0.22783    -0.23501E-01  0.13936E-02  -0.33083E-04      9
   1.772 E-03  288  2.039 E-03    512  1.031 E-07    576
   273   576
  0.11008E 03-0.14339E 02 0.69993    -0.15178E-01  0.12334E-03                   10
```

IV. Sample Problem Input Case II

```
ZPPR-5  PHASE A  Na VOIDED  AXIAL 1  MASTER  05-1-185
    10    576    -2     1
    -4     -5     6     5     6     4     -5     -4     -5     -4
 .776       0.0031248     1030.08
 304 SSA 304 SSB    U-235  Pu-240   U-238  Pu-239 SODIUM1 SODIUM2 OXYGEN1 OXYGEN2
 1.0          1.0          1.0685      0.9831      1.0333      1.0282
 1.0          1.0          1.0         1.0
     0     0       0
  3.366 E-03    288   3.041 E-03   512  6.929 E-03   576
     1   288
-3.42071       .19897E-01   .31268E-01 -.87926E-03                              1
 3.366  E-03    288   3.041 E-03   512  6.929 E-03   576
   289   576
 -.64550E 01  .20166E 01  -.14562       .39947E-02  -.37783E-04                 2
  2.399 E-05    288   3.038 E-05   512    0.0        576
  .86659E 02  .19268E 01  -.69235       .32865E-01  -.6382 E-03  .46582E-05     3
  2.260 E-04    288     0.0        512    0.0        576
  .13955E 02 -.82689      .1027        -.82339E-02   .1861 E-03                 4
  1.116 E-02    288   1.416 E-02   512    0.0        576
-.42195 E 01 -.36142      .96710E-01   -.52222E-02   .1079E-03  -.75529E-06     5
  1.725 E-03    288     0.0        512    0.0        576
  .11189E 03  .14994    -.47943        .12386E-01                              6
     0.0        288     0.0        512  2.076 E-03   576
     1   272
-0.60308E 01 -0.27176   0.14310       -0.1458E-01  0.86712E-03 -0.20683E-04     7
     0.0        288     0.0        512  2.076 E-03   576
   273   576
 0.59634E 02 -0.72497E 01 0.33290    -0.68368E-02  0.52934E-04                 8
  1.1363E-03    288   2.0388E-03   512   9.375E-08   576
     1   272
-0.98429E 01 -0.44102     0.22783    -0.23501E-01  0.13936E-02 -0.33083E-04     9
  1.1363E-03    288   2.0388E-03   512   9.375E-08   576
   273   576
 0.11008E 03 -0.14339E 02 0.69993    -0.15178E-01  0.12334E-03
```

V. Sample Problem Input Case III

```
ZPPR-5  PHASE A  AXIAL-1  STEEL SLUMPED  MASTER  05-1-201
    10   576    -2     1
    -6    -5     6     5     6     6     -5     -4     -5     -4
 .776      0.0031248     1030.08
 304 SSA 304 SSB   U-235  Pu-240   U-238   Pu-239 SODIUM1 SODIUM2 OXYGEN1 OXYGEN2
 1.0         1.0         1.0685      0.9831        1.0333     1.0282
 1.0         1.0         1.0         1.0
     0     0     0
  1.125 E-03   224 1.120  E-02   288  3.041 E-03    512 6.929  E-03    576
     1   288
 -.35602E 01 -.72695       .28571      -.35759E-01   .21442E-02  -.47891E-04       1
  1.125 E-03   224 3.041  E-03   512  6.929 E-03    576
   289   576
  .45495E 02 -.58699E 01  .29106      -.64907E-02   .54344E                        2
  2.399 E-05   288 3.038  E-05   512    0.0         576
  .86659E 02  .19268E 01 -.69235       .32865E-01  -.63819E-03   .46582E-05       3
  2.272 E-04   224 2.219  E-04   288    0.0         576
  .13955E 02 -.82689       .10270      -.82339E-02   .18606E-03                   4
  1.116 E-02   288 1.416  E-02   512    0.0         576
 -.38588E 01 -.40926       .10633      -.60430E-02   .13500E-03  -.10572E-05      5
  1.725 E-03   288                 576
  .11138E 03  .72565     -.66209        .53677E-01  -.30013E-02   .66037E-04       6
   0.0         512  2.076 E-03   576
     1   272
-0.60308E 01-0.27176     0.14310      -0.1458E-01   0.86712E-03  -0.2068E-04      7
   0.0         512  2.076 E-03   576
   273    576
  0.59634E 02-0.72497E 01 0.33290     -0.68368E-02 0.52934E-04                    8
   1.1363E-03  288  2.0388E-03   512    9.375E-08   576
     1   272
 -0.98429E 01-0.44102     0.22783     -0.23501E-01 0.13936E-02 -0.33083E-04       9
   1.1363E-03  288  2.0388E-03   512    9.375E-08   576
    273   576
  0.11008E 03-0.14339E 02 0.69993     -0.15178E-01 0.12334E-03                    10
```

VI. Sample Problem Input Case IV

```
ZPPR-5  PHASE A  AXIAL 1  FUEL SLUMPED  MASTER  05-1-211
    10   576    -2     1
    -5    -5     5     6     6     6     -5     -4     -5     -4
 .776       0.0031248      1030.08
 304 SSA 304 SSB   U-235  Pu-240   U-238   Pu-239 SODIUM1 SODIUM2 OXYGEN1 OXYGEN2
 1.0           1.0          1.0685     0.9831        1.0333       1.0282
 1.0           1.0          1.0        1.0
     0     0     0
  5.761 E-04    112 1.346  E-03    224 1.121  E-02    288   0.0          576
     1   288
 -.48891E 01  .23935E 01 -.61383       .53279E-01  -.14118E-02                    1
  5.761 E-04    112 1.346  E-03    224 1.121  E-02    288   0.0          576
   289   576
 -.33756E 02  .61941E 01 -.38163       .98611E-02  -.92146E-04                    2
   0.0          112 4.804  E-05    224 2.382  E-05    288  3.038 E-05    576
  .51977E 02  .67554E 01 -.87331       .28746E-01  -.29966E-03                    3
   0.0          112 4.544  E-04    224  2.219 E-04    288  0.0           576
  .14881E 02 -.32618E 01  .84897      -.75508E-01   .25651E-02 -.28974E-04        4
   0.0          112 2.235  E-02    224  1.108 E-02    288  1.416 E-02    576
 -.33876E 01 -.84358       .15716      -.83396E-02   .18100E-03 -.14040E-05        5
   0.0          112 3.466  E-03    224  1.696 E-03    288  0.0           576
  .11084E 03 -.17714E 02  .51261 01  -.55018         .23149E-01 -.33979E-03        6
     0.0        288       0.0       512  2.076 E-03    576
    1   272
-0.60308E 01-0.27176      0.14310      -0.1458E-01  0.86712E-03-0.20683E-04        7
     0.0        288       0.0       512  2.076 E-03    576
  273   576
 0.59634E 02-0.72497E 01 0.33290     -0.68368E-02  0.52934E-04                    8
     0.0        112  2.267 E-03    224  1.136 E-03    288  2.039 E-03    576
    1   272
-0.98429E 01-0.44102      0.22783     -0.23501E-01  0.13936E-02-0.33083E-04        9
     0.0        112  2.267 E-03    224  1.136 E-03    288  2.039 E-03    576
  273  576
 0.11008E 03-0.14339E 02 0.69993     -0.16178E-01  0.12334E-03                   10
```

APPENDIX F

Method of Least Squares*

Our data consists of pairs of measurements (x_i, y_i) of an independent variable x and a dependent variable y. We wish to fit the data with an equation of the form

$$y = a + b\,x$$

by determining the values of the coefficient a and b such that the discrepancy is minimized between the values of our measurements y_i, and the corresponding values

$$y = f(x_i)$$

on the fitted line.

For any arbitrary values of a and b, we can calculate the deviations Δy_i between each of the observed values y_i and the corresponding calculated values

$$\Delta y_i = y_i - a - b\,x_i$$

If the coefficients are well chosen, these deviations are small. The sum of these deviations are not a good measure of how well the data is approximated with our fitted line because large positive deviations can be balanced by large

negative ones to yield a small sum even when the fit is bad. We consider the sum of the squares of deviations. The quantity X^2 is defined by the sum

$$X^2 = \sum \left(\frac{\Delta y_i}{\sigma_i}\right)^2 = \sum \left[\frac{1}{\sigma_i^2} (y_i - a - b\, x_i)^2\right]$$

For any given value of

$x = x_i$

y_i = observed measurement

σ_i = standard deviation for the observations about the actual value $y(x_i)$.

The method for finding the optimum fit to the data will be to minimize the weighted sum of squares of deviations X^2. The minimum value of the function X^2 is one which yields a value of 0 to both of the partial derivatives with respect to each of the coefficients

$$\frac{\partial}{\partial_a} X^2 = \frac{\partial}{\partial_a} \left[\frac{1}{\sigma^2} \sum (y_i - a - bx_i)^2\right] = \frac{-2}{\sigma^2} \sum (y_i - a - bx_i) = 0$$

$$\frac{\partial}{\partial_b} X^2 = \frac{\partial}{\partial_b} \left[\frac{1}{\sigma^2} \sum (y_i - a - bx_i)^2\right] = \frac{-2}{\sigma^2} \sum [x_i (y_i - a - bx_i)] = 0$$

These equations can be rearranged to yield a pair of simultaneous equations

$$\sum y_i = aN + b \sum x_i$$

$$\sum x_i y_i = a \sum x_i + b \sum x_i^2$$

Solving these equations for a and b will give the values of the coefficients for which the sum of squares of the deviations of the data points from the calculated fit (X^2), is a minimum.

The data from this experiment were not fit well by a straight line so one constructs a more complex function which includes curvature and try varying the coefficients of this function to fit the data more closely. The same method to optimize the values of the coefficients of this function to fit the data more closely. The same method is optimize the values of the coefficients a and b in the fit to a straight line outlined above can be applied to fitting a higher order polynomials to the data.

To illustrate the method, the data can be fixed with a quadratic curve

$$y = a + b\,x + c\,x^2$$

then the results can be extrapolated to include polynomials of any order

$$X^2 \equiv \sum \left(\frac{\Delta y_i}{\sigma_i}\right)^2 = \sum \left[\frac{1}{\sigma_i^2}\,(y_i - a - bx_i - cx_i^2)^2\right]$$

Setting the derivatives of X^2 with respect to each of the three coefficients a, b, and c equal to 0 yields three simultaneous equations:

$$\sum y_i = a \sum 1 + b \sum x_i + c \sum x_i^2$$

$$\sum x_i y_i = a \sum x_i + b \sum x_i^2 + c \sum x_i^3$$

$$\sum x_i^2 y_i = a \sum x_i^2 + b \sum x_i^3 + c \sum x_i^4$$

with a factor σ_i^2 in the denominator of each term inside each summation sign.

These simultaneous equations can be solved by determinant methods but it can become very cumbersome when used for polynomials of order 3 or more. The computer program used to fit the data obtained in this experiment uses a matrix inversion method to solve for the coefficients (see chapter 8 in ref. Bevington, Philip R., Data Reduction and Error Analysis for the Physical Sciences, McGraw-Hill, Inc., New York, NY, 1969).

VITA

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